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ABSTRACT

This monograph is the first in a series of three dealing with the curriculum and evaluation standards for grades K-4 as promulgated by the National Council of Teachers of Mathematics (NCTM). The publication of the standards marks a new guidepost in the evolution of mathematics education and signals the impact of the information society upon all citizens through the recognition of the importance of the mathematical skills involving problem solving, estimation, reasonableness of computed results, and use of technology, among others. Included in this first volume are: (1) introductory information concerning the mathematics essential to 21st century citizens as well as the summary changes in the K-4 curriculum; (2) essays, sample problems, and activities dealing with Standard #1, problem solving; (3) activities and real-world applications revolving around Standard #2, mathematical communication; (4) sample problems and activities concerning Standard #3, mathematical reasoning; (5) reflections and activities involving Standard #4, mathematical connections; (6) essays, examples, and activities affecting Standard #5, mathematical estimation; and (7) appendices that contain the NCTM position statements pertaining to both calculator usage and mathematics leadership within the elementary school venue and suggestions for curriculum development and staff implementation. (JJK)

****
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A publication of the Virginia Council of Teachers of Mathematics

Implementing the Mathematics Standards

Professor Bear - Special Agent

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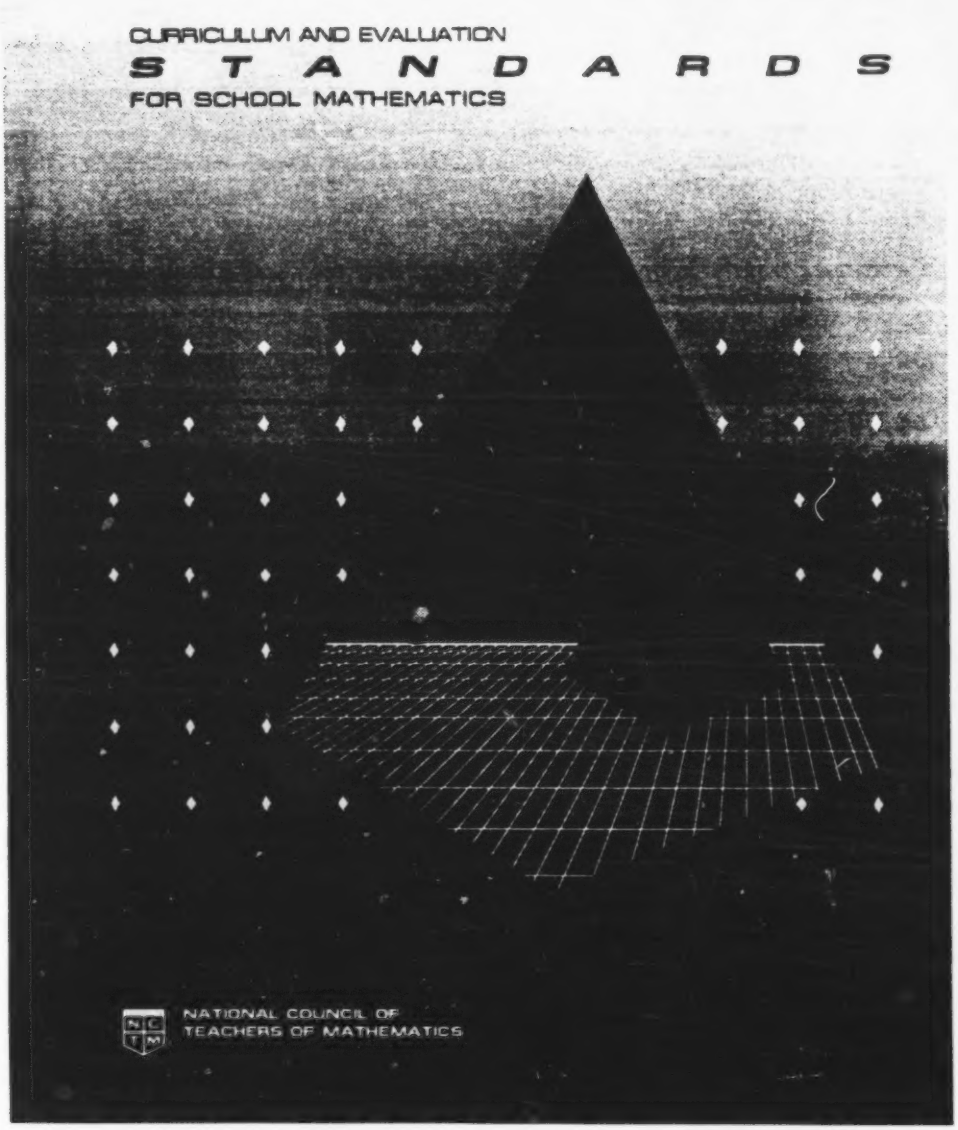

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A Monograph of the Virginia Council of Teachers of Mathematics

Volume 1 **Fall 1989**

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Prologue

Bob Underhill

This monograph is the first in a series of two or three which will be published this year by the Virginia Council of Teachers of Mathematics for all persons interested in the mathematical development of children in grades kindergarten through four. It is our sincere hope that these monographs will stimulate much discussion and reflection.

The publication of the Standards marks a new guidepost in the evolution of mathematics education. We have moved from a society dominated by agrarian and small business needs into an information society. With this change, we find concomitant needs for problem solving, estimation, reasonableness of results, use of technology and so on in the lives of every person.

In national and international studies of mathematics achievement, United States' students keep falling at the bottom of the scale in comparisons of achievement across years and across first world countries. For two decades we attempted to explain away many of these differences by arguing that the items were inappropriate or that the populations were not comparable. As the testing has become more sophisticated in responding to these issues, U.S. learners have continued to be at the bottom.

There is a challenge to all of us as students, parents, teachers and college faculty to engage in discussion and reflection about what we are doing and how we can change what we are doing in ways which will enhance learners' mathematical knowledge. The Standards have gone through a three-year development effort by many of our best and most creative professionals. The document is not "theirs;" it is OURS.

The first five Standards are presented in this monograph. They attend to five thematic areas which deserve our thought and deliberation. What we sincerely hope is that we have provided considerable examples which will help all interested persons to not only *understand*, but also to begin taking steps which will move toward implementation. VCTM wishes you well on your journey. Please communicate with me on behalf of VCTM publications if you like what we have prepared and if you have any issues or concerns to be addressed which you envision to which these monographs might not attend. My address is printed inside the front cover.

Notes

Before signing off, I'd like to mention a few items of public note:

1. We are all indebted to Jim N. Boyd of the St. Christophers School mathematics department in Richmond for the bear art throughout the monographs.

2. We are indebted to several authors whose work appeared in the publications of other states' councils. They have been noted at the end of such articles.

3. We hope this monograph will *not* be viewed as a substitute for the Standards. The comprehensive document is available through NCTM, and ordering information is available inside the front and back covers.

4. VCTM welcomes your participation in the Council as an individual or as a school. Membership is only $10 each year. It includes the journal, published three times each year, and reduced registration at the annual spring meeting (see back cover for dates and locations).

5. The Standards are not discrete. You will find that the activities we've suggested often address 2, or even 3, Standards at once.

6. We have additional copies of this monograph. They may be ordered from me for $2.50 for schools and $2.61 for individuals.

7. Finally, we would welcome you most heartedly to participate in this venture as an AUTHOR. Please mail manuscripts to me no matter how short!

Bob is a math educator at Virginia Tech.

WHY STUDENTS HATE MATHEMATICS

by Lynn Oberlin

(The following article has been reprinted from the Minnesota Council of Teachers of Mathematics Newsletter, March, 1983)

Children generally do not hate Mathematics when they start school. This is a trait which they acquire as part of their elementary school training.

The feat of loathing mathematics can generally be accomplished if the teacher will use one or more of the following procedures:

1. Assign the same work to everyone in the class. This technique is effective with about two-thirds of the frustrated from trying to do the impossible while the top third will hate the boredom. Warning: This may not be effective with about the middle 1/3 of the students.

2. Go through the book, problem by problem, page by page. In time, the drudgery and monotony is bound to get to them.

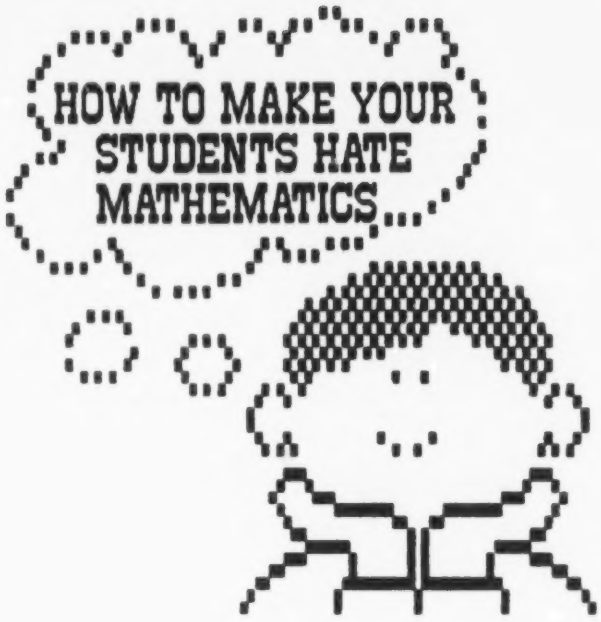
3. Assign written work everyday. Before long, just the word "mathematics" will remove every smile in the room.

4. Be sure that each student has plenty of homework. This is especially important over the weekends and vacation periods.

5. Never correlate Mathematics with life situations. A student might find it useful and get to enjoy Math.

6. Insist there is ONLY one correct way to solve each problem. This is very important as some creative student might look for different ways to solve a problem. He could even grow to like Math.

7. Assign Mathematics as punishment for misbehavior. The association works wonders. Soon math and punishment will take on the same meaning.



8. Be sure that ALL students complete ALL the review work in front of the texbook. This ought to last until Thanksgiving or Christmas, and is certain to kill off the interest of most students.

9. Use long drill type assignments with many examples of the SAME type problem. (For example: 30 long column addition problems). This type of assignment requires little teacher time and keeps the student occupied for a long time. The majority of pupils are sure to dislike it.

10. Always insist that the papers are prepared a certain way. Name, date, page number, etc., must each be placed in a specific spot. If a students fails to follow this procedure, tear up his paper and let him start over again. Instant humiliation and despair are almost guaranteed.

11. Lastly, insist that EVERY problem worked incorrectly be reworked until it is correct. This procedure is most effective in promoting distaste for math and if followed very carefully, the student may even learn to detest his teacher as well!

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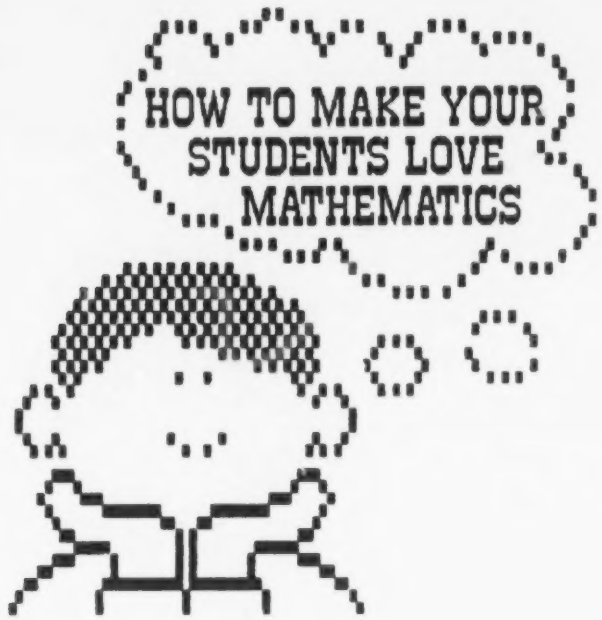

WHY STUDENTS LOVE MATHEMATICS

by Leslie Stadd

After reading Lynn Oberlin's article, I couldn't help but think of several teachers I've observed who practiced those skills with such finesse. It soon became clear to me that the teachers did not know any better - afterall, it was the way their teachers had taught them. Sad, but so true! Within their classrooms there is no life, no laughter, no love, and often, no learning! What is real unfortunate is that these teachers perform quite well on teacher performance appraisals, so they are here to stay.

Somewhat depressed that these people are teaching our children, I figured what was needed was a dose of Max Sobel. Max Sobel is not a milk of magnesia, or caster oil type cure. It is a source of revitalization that can be found in a book entitled TEACHING MATHEMATICS: A Sourcebook of Aids, Activities, and Strategies written by Sobel and Evan M. Maletsky.

The book is filled with ideas that can be used in the classroom to stimulate the students' love for mathematics. Summarizing the five "musts", the teacher must:

1. ASK INTERESTING AND EXCITING QUESTIONS

How long would it take me to count to a million one number per second?

2. MAKE PROVISIONS FOR DISCOVERY

What is the sum of the whole numbers from 1 to 100 inclusive?

3. MAKE MATHEMATICS COME ALIVE

Why not bring Karl Gauss into your class? He is said to have answered the above problem in a flash as an elementary student.

4. START OR END THE PERIOD WITH SOMETHING SPECTACULAR

How can you prove 1 = 2?

5. MAKE EFFECTIVE USE OF MULTISENSORY AIDS

It's hard to believe how a simple piece of string can be use to teach a variety of topics!

I was inspired by the book and it allowed me the chance to try many things in my classroom. It wasn't long before it was obvious my students enjoyed coming to my class and would moan when they heard the bell to dismiss. The feeling was well worth making the change in my behavior. I added enthusiasm, and it is contagious! I was one of those teachers mentioned in Lynn Oberlin's article, but never again.

If you know of a teacher who makes mathematics miserable, please xerox this article and get it to him/her (perhaps discreetly) and maybe you can make a difference. Perhaps someone else might be able to say:

THANKS MAX, I NEEDED THAT!

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Essential Mathematics for the 21st Century

The Position of The National Council of Supervisors of Mathematics, June 1988

...The NCSM position statement of 1977 responded to the "Back to the Basics" movement with its overly-narrow conception of basic skills. Now, as we look to the future, we recognize that the use of calculators and computers and the application of statistical methods will continue to expand. Creative problem solving, precise reasoning, and effective communication will grow in importance. To function effectively in the next century, students will need proficiency in an enriched body of essential mathematics. The list that follows identifies twelve critical areas of mathematical competence for all students. It does not imply an instructional sequence or a priority among topics. In fact, the twelve essential mathematics areas are interrelated; competence in each area requires competence in other areas.

PROBLEM SOLVING. Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-text problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. Students should see alternate solutions to problems; they should experience problems with more than a single solution.

COMMUNICATING MATHEMATICAL IDEAS. Students should learn the language and notation of mathematics. For example, they should understand place value and scientific notation. They should learn to receive mathematical ideas through listening, reading, and visualizing. They should be able to present mathematical ideas by speaking, writing, drawing pictures and graphs, and demonstrating with concrete models. They should be able to discuss mathematics and ask questions about mathematics.

MATHEMATICAL REASONING. Students should learn to make independent investigations of mathematical ideas. They should be able to identify and extend patterns and use experiences and observations to make conjectures (tentative conclusions). They should learn to use a counter-example to disprove a conjecture and they should learn to use models, known facts, and logical arguments to validate a conjecture. They should be able to distinguish between valid and invalid arguments.

ALERTNESS TO THE REASONABLENESS OF RESULTS. In solving problems, students should question the reasonableness of a solution or conjecture in relation to the original problem. Students must develop the number sense to determine if results of calculations are reasonable in relations to the original numbers and the operations used. With the increase in the use of calculating devices in society, this capability is more important than ever.

ESTIMATION. Students should be able to carry out rapid approximate calculations through the use of mental arithmetic and a variety of computational estimation techniques. When computation is needed in a problem or consumer setting, an estimate can be used to check reasonableness, examine a conjecture, or make a decision. Students should acquire simple techniques for estimating measurements such as length, area, volume, and mass (weight). They should be able to decide when a particular result is precise enough for the purpose at hand.

APPROPRIATE COMPUTATIONAL SKILLS. Students should gain facility in using addition, subtraction, multiplication, and division with whole numbers and decimals. Today, long, complicated computations should be done with a calculator or computer. Knowledge of single-digit number facts is essential, and using mental arithmetic is a valuable skill. In learning to apply computation, students should have practice in choosing the appropriate computational method: mental arithmetic, paper-pencil algorithm, or calculating device. Moreover, there are everyday situations that demand recognition of, and simple computation with, common fractions. In addition, the ability to recognize, use, and estimate with percents must also be developed and maintained.

ALGEBRAIC THINKING. Students should learn to use variables (letters) to represent mathematical quantities and expressions; they should be able to represent mathematical functions and relationships using tables, graphs, and equations. They should understand and correctly use positive and negative numbers, order of operations, formulas, equations and inequalities. They should recognize the ways in which one quantity changes in relation to another.

MEASUREMENT. Students should learn the fundamental concepts of measurement through concrete experiences. They should be able to measure distance, mass (weight), time, capacity, temperature, and angles. They should learn to calculate simple perimeters, areas, and volumes. they should be able to perform measurement in both metric and customary systems using the appropriate tools and levels of precision.

GEOMETRY. Students should understand the geometric concepts necessary to function effectively in the three-dimensional world. They should have knowledge of concepts such as parallelism, perpen-

dicularity, congruence, similarity and symmetry. Students should know properties of simple plane and solid geometric figures. Students should visualize and verbalize how objects move in the world around them using terms such as slides, flips, and turns. Geometric concepts should be explored in settings that involve problem solving and measurement.

STATISTICS. Students should plan and carry out the collection and organization of data to answer questions in their everyday lives. Students should know how to construct, read, and draw conclusions from simple tables, maps, charts, and graphs. They should be able to present information about numerical data such as measures of central tendency (mean, median, mode) and measures of dispersion (range, deviation). Students should recognize the basic uses and misuses of statistical representation and inference.

PROBABILITY. Students should understand elementary notions of probability in determining the likelihood of future events. They should identify situations where immediate past experience does not affect the likelihood of future events. They should become familiar with how mathematics is used to help make predictions such as election results, business forecasts, and outcomes of sporting events. They should know that probability applies to research results and to the decision making process.

Climate for Learning

To learn the essential mathematics needed for the 21st century, students need a nonthreatening environment in which they are encouraged to ask questions and take risks. The learning climate should incorporate high expectations for all students, regardless of sex, race, handicapping condition, or socioeconomic status. Students need to explore mathematics using manipulatives, measuring devices, models, calculators, and computers. They need to have opportunities to talk to each other about mathematics.

Students need modes of instruction that are suitable for the increased emphasis on problem solving, applications, and higher-order thinking skills. For example, cooperative learning allows students to work together in problem-solving situations to pose questions, analyze solutions, try alternative strategies, and check for reasonableness of results.

To implement the new instructional strategies, extensive professional development opportunities as well as new learning materials will be needed.

Technology

Calculators should be used by students throughout the mathematics program, beginning in the primary grades. As adults, students will use calculators or computers to do difficult computations. They will need facility with single-digit facts, estimation skills, and mental arithmetic, and they must be able to determine if results obtained from calculators or computers are reasonable. Students will need practice in deciding if calculations should be done mentally, with paper and pencil, or with a computing device.

The use of computers should be incorporated throughout the K-12 mathematics curriculum. Mathematics classrooms should be equipped with computers and projection devices or multiple large-screen monitors for classroom demonstrations. In addition, computer laboratories should be readily available to all students. Computer use should move away from drill and practice on isolated low-level skills and toward meaningful involvement by students in problem solving and concept development.

Telecommunication technology should be used to make opportunities to learn mathematics available to all students in all locations.

Evaluation

Evaluation at each administrative level, from the state or province level to the classroom, should be aligned with the objectives of the curriculum. At this time, caution should be exercised in using standardized tests to monitor student progress and evaluate the effectiveness of instruction. Existing standardized tests could perpetuate the domination of the mathematics curriculum by lower-order skills. To prepare for the 21st century, there is a need for new tests that shift the focus from computation to problem solving and reasoning. Use of calculators should be allowed on tests. In addition to paper-and pencil tests, evaluation should involve other means such as teacher observations, interviews, student projects, and presentations.

Extending the Essentials

As we move from the industrial society of the 20th century to the information society of the 21st century, knowledge of mathematics is becoming increasingly important for individuals who wish to have options for careers and higher education. Almost all careers require a background in mathematics. Most college majors require elementary algebra, advanced algebra, and geometry as prerequisites. In addition, students specializing in fields like engineering, the sciences, or business will need a precalculus course that includes trigonometry. Today most majors involve some statistics. Opportunities to learn statistics, probability, discrete mathematics, and computer courses should be available in high school. Students who have completed a precalculus course by their senior year should have the opportunity to take a college-level calculus or a discrete mathematics course in their senior year. Students should plan to take mathematics every year, avoiding gaps in their mathematics education that could require remediation when they resume their study of mathematics. The more mathematics students take in high school, the more options they will have for the future.

Today our society has inequities by sex and race in employment, income, and participation in mathematics. To move toward an equitable society in the next century, we must address the disparities in mathematics education in this century. Working together with high expectations for all, we will offer our students the education that will provide pathways to opportunity in the 21st century.

SUMMARY OF CHANGES IN CONTENT AND EMPHASIS IN K–4 MATHEMATICS

INCREASED ATTENTION

NUMBER
- Number sense
- Place-value concepts
- Meaning of fractions and decimals
- Estimation of quantities

OPERATIONS AND COMPUTATION
- Meaning of operations
- Operation sense
- Mental computation
- Estimation and the reasonableness of answers
- Selection of an appropriate computational method
- Use of calculators for complex computation
- Thinking strategies for basic facts

GEOMETRY AND MEASUREMENT
- Properties of geometric figures
- Geometric relationships
- Spatial sense
- Process of measuring
- Concepts related to units of measurement
- Actual measuring
- Estimation of measurements
- Use of measurement and geometry ideas throughout the curriculum

PROBABILITY AND STATISTICS
- Collection and organization of data
- Exploration of chance

PATTERNS AND RELATIONSHIPS
- Pattern recognition and description
- Use of variables to express relationships

PROBLEM SOLVING
- Word problems with a variety of structures
- Use of everyday problems
- Applications
- Study of patterns and relationships
- Problem-solving strategies

INSTRUCTIONAL PRACTICES
- Use of manipulative materials
- Cooperative work
- Discussion of mathematics
- Questioning
- Justification of thinking
- Writing about mathematics
- Problem-solving approach to instruction
- Content integration
- Use of calculators and computers

DECREASED ATTENTION

NUMBER
- Early attention to reading, writing, and ordering numbers symbolically

OPERATIONS AND COMPUTATION
- Complex paper-and-pencil computations
- Isolated treatment of paper-and-pencil computations
- Addition and subtraction without renaming
- Isolated treatment of division facts
- Long division
- Long division without remainders
- Paper-and-pencil fraction computation
- Use of rounding to estimate

GEOMETRY AND MEASUREMENT
- Primary focus on naming geometric figures
- Memorization of equivalencies between units of measurement

PROBLEM SOLVING
- Use of clue words to determine which operation to use

INSTRUCTIONAL PRACTICES
- Rote practice
- Rote memorization of rules
- One answer and one method
- Use of worksheets
- Written practice
- Teaching by telling

STANDARD 1:
MATHEMATICS AS PROBLEM SOLVING

In grades K-4, the study of mathematics should emphasize problem solving so that students can—

- ***use problem-solving approaches to investigate and understand mathematical content;***
- ***formulate problems from everyday and mathematical situations;***
- ***develop and apply strategies to solve a wide variety of problems;***
- ***verify and interpret results with respect to the original problem;***
- ***acquire confidence in using mathematics meaningfully.***

Focus

Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.

This standard emphasizes a comprehensive and rich approach to problem solving in a classroom climate that encourages and supports problem-solving efforts. Ideally, students should share their thinking and approaches with other students and with teachers, and they should learn several ways of representing problems and strategies for solving them. In addition, they should learn to value the process of solving problems as much as they value the solutions. Students should have many experiences in creating problems from real-world activities, from organized data, and from equations.

In the early years of the K–4 program, most problem situations will arise from school and other everyday experiences. When mathematics evolves naturally from problem situations that have meaning to children and are regularly related to their environment, it becomes relevant and helps children link their knowledge to many kinds of situations. As children progress through the grades, they should encounter more diverse and complex types of problems that arise from both real-world and mathematical contexts.

When problem solving becomes an integral part of classroom instruction and children experience success in solving problems, they gain confidence in doing mathematics and develop persevering and inquiring minds. They also grow in their ability to communicate mathematically and use higher-level thinking processes.

Elementary Problem Box

86-13

1989

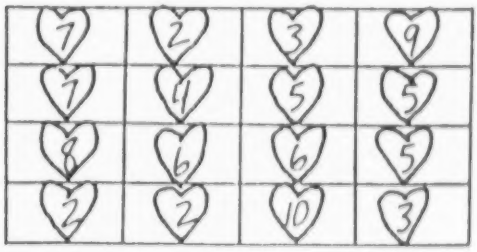
Use the numbers listed to make the sum of 1987 in every horizontal, vertical and diagonal row of four. You may use a calculator to help you.

60
125
292
357
473
643
813
873
929

243	525		
	408		
		581	178
	699		757

86-14 Circle sums for 14. The numbers must be adjoining and you must circle every number.

EXAMPLE:



86-15 Five people – Jo Ann, Rob, Julie, Randy and Fred are building a snowfort. Which snowball belongs to each person.

If...
- Jo Ann's snowball touches Fred and Randy's
- Randy's touches Rob's and Jo Ann's
- Julie's is on the left
- Rob's is not on the far right

Label each snowball correctly.

86-16 Continue this pattern. Include all necessary details.

Used with permission - The Oregon Mathematics Teacher (1987)

Teaching Problem Solving in the Early Grades

Mark A. Spikell

The National Council Teachers of Mathematics (NCTM) curriculum standards state that "In grades K-4, problem solving should permeate the mathematics curriculum so that students can:

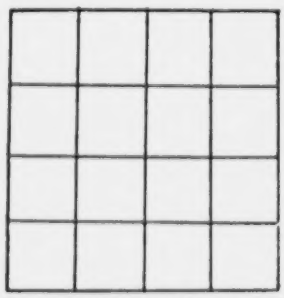
- use problem solving processes in their learning of all mathematics content;
- use strategies in solving a wide variety of problems from many contexts;
- discuss alternate solution strategies and relationships among problems;
- formulate problems." (1)

To help students achieve these goals, learners need learning environments which enable them to do problem solving. And, "to do problem solving" means that students need to engage in the problem solving steps first articulated by Polya and nicely summarized in (2). They are:

1. See
2. Plan
3. Do
4. Check

But how does one create learning environments for students to experience the four problem solving steps and the goals called for by the NCTM? One method is suggested in this article. Through a discussion of a specific problem and the technique useful to present, solve and discuss it with fourth graders, we will examine some ideas on how we can create learning environments which encourage and develop student problem solving.

Pose the question in Figure 1, preferably with pairs or small groups of learners or as a homework problem. Ask students to write their answers on a scrap of paper. Invite two students to collect the scraps, organize the data and record it on the chalkboard. Figure 2 shows the type of data you might expect.



How many squares do you see?

FIGURE 1

Number of Squares	Number of teams or respondents
16	5
17	3
19	1
21	1
23	1
30	2

FIGURE 2

Usually, most students will provide plausible as opposed to implausible answers. Plausible answers, such as those in Figure 2, are ones which have been obtained by some logical, explainable means. For example, a student who offered the answer of seventeen (17) might explain that she "....counted all of the sixteen (16) small squares and the one (1) large square too...." And, another student who offered the answer of twenty-one (21) might explain that he "....counted all of the small squares, the large square, and the four corners....," as shown in Figure 3.

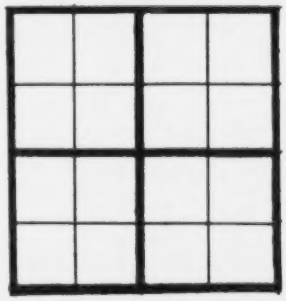


FIGURE 3

Implausible answers are guesses or clearly unrealistic responses. For example, implausible answers might include zero (0) or one million (1,000,000) or five-and-one-third (5⅓). One can see some squares so zero is unrealistic. One million is a very large number and the relative size and subdivisions of the grid are quite small. Thus, one million is an unlikely answer. Finally, squares are counted as "whole" objects so counting fractional squares makes no sense in this context.

Once the data in Figure 2 are on the chalkboard, the teacher leads a class discussion to (a) eliminate implausible answers and (b) obtain explanations for all plausible answers. A good technique to use is to ask teams or individual students to explain to the class how they arrived at the answers given. Most students enjoy talking about their work. With some help from the teacher, this usually leads to most students understanding why some answers are implausible and to general agreement on how various students obtained the plausible answers.

Once the plausible answers are established, the teacher should make an effort to point out that the problem, as stated, does have many possible, plausible answers. Then, the teacher can ask the students how the original problem could be changed so there could be only one correct (plausible) answer. And, further, what do students think is the best, most interesting question to ask? Some rephrased responses from students might include:

- How many small (1x1) squares are there? (Answer: 16)
- How many non-overlapping squares are there? (Answer: 22)
- How many total squares of all sizes are there? (Answer: 30)

•How many different sizes of squares are there? (Answer: 4)

Generally, after suitable discussion, many students may conclude that asking the question, "How many total squares are there, including overlapping squares," is the best, most interesting question to ask. They may feel this way because the answer of thirty (30) is the largest of the plausible answers, yet small enough for virtually all students to count and verify once they understand that there are several sizes of squares (1x1, 2x2, 3x3, and 4x4) and that both overlapping and non-overlapping squares can be included.

As a follow-up to the above problem exploration, perhaps as homework, teachers should pose the question of "How many total squares are there for a 5x5 grid?" as shown in Figure 4. Or, they might ask "How many triangles are in Figure 5?"

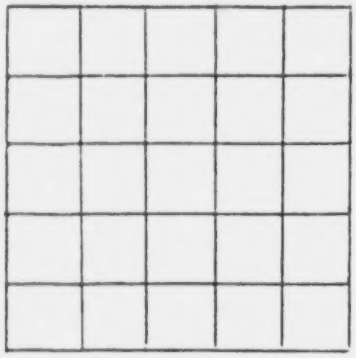


FIGURE 4

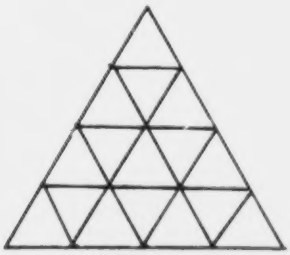


FIGURE 5

If the problem and technique of presentation described above are used, students have opportunities to perform or practice all of the four steps in Polya's problem-solving process. This in turn enables them to experience the four goals cited in the NCTM standards.

In initially attempting to solve the problem and then participating in a discussion of plausible and implausible answers obtained by classmates, students can experience first-hand the importance of understanding what question is to be answered, why some strategies work and others do not, and what it means for an answer to be reasonable in the context of the problem under exploration.

Then, when students solve the problem extension (the question for the 5x5 square grid or the 4x4 triangular grid), they have an opportunity to revisit the various definitions, solution techniques and insights provided in the original problem exploration (the 4x4 grid). Frequently, it is during this revisitation step when many students actually learn or internalize key ideas associated with problem activity. This is particularly true when teachers spend some time reviewing the answer to the extension problem and helping students focus on the key ideas embodied in the problem and its solution. For instance, some of the central ideas in this problem exploration might include:

- •What is the meaning of overlapping and non-overlapping squares?
- •How does one determine the total number of sizes of squares which could be counted?
- •How does one count squares in an organized way so as not to miss any?
- •How do the techniques discussed apply to other similar problems on square and non-square grids?

Posing problems that do not have an obvious single interpretation is extremely valuable when teaching problem solving. As illustrated in the preceding discussion, this enables you to encourage students to think about how to restate problems so they are not ambiguous; and, this is one important aspect of helping students learn how to formulate problems, one of the goals specifically cited in the NCTM standards.

In closing, it seems useful to suggest that the method described here — pose an interesting but ambiguous problem, engage students as a group in exploring the problem, provide students with a directly related extension or follow-up problem - can be used by many teachers as a general approach to create an environment in which students can learn, by doing, mathematical problem solving. Increasingly, mathematics textbooks and supplementary materials for teachers are including many problems of the type suggested here (see, for example, (2)). All that remains is for us to modify the way in which we present problems by exploring and discussing them with students. If we do this, it is likely that we will have a profound impact on helping students become good problem solvers.

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(2) *Problem Solving...a basic mathematics goal. Becoming a better problem solver 1.* Ohio Department of Education, Columbus, Ohio, 1980, pg. 21-23. (Available from Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303).

Mark is a member of the George Mason University.

A Day At The Park

By: Rick Farrell

NUMBER OF PLAYERS:

Two players or two teams

OBJECTIVES:

To match rods and numerals

GRADE LEVEL:

K-2

MATERIALS:

Each player receives a die, a set of the six shortest Cuisenaire Rods, and a copy of the game board.

PROCEDURE:

Each player writes his/her name next to the house selected on the game board. Each then rolls the die to see who goes first. The numbers appearing on the die are matched to the rods. For example, a three is matched to the light green rod, etc. Using the rods, the player tries to get to the park by building a path of rods corresponding to the number on each roll of the die. The first person to reach the park wins. If a rod is too long for the player to remain on the path, the player can choose two shorter rods that equal the rod selected on this roll. for example, a red rod + a light green equals a yellow rod.

PURPOSE:

This game serves as an introduction to addition and number recognition for primary students. As the game is being played, the children should recognize the numerical values of the rod and say the equations out loud when they combine two rods to equal one.

Professor Bear has an idea concerning the revision of the math curriculum?

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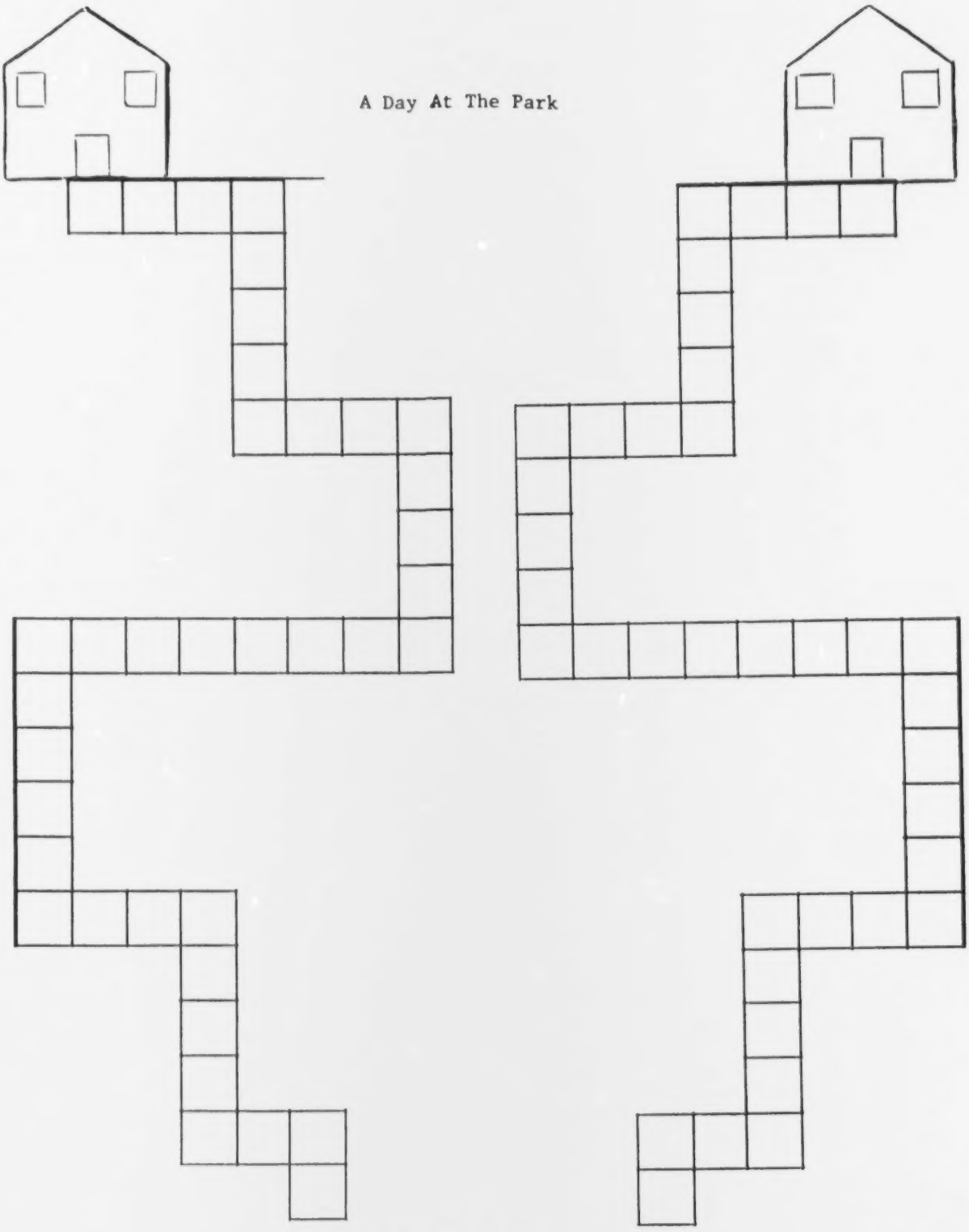
A Day At The Park



Cuisenaire Park

Everyday Mathematics: Learning From Real-Life Experiences

Paul Dobransky
Marple Newtown School District

Jean Kerrigan
West Chester University

John Kerrigan
West Chester University

Judy Stopper
Marple Newtown School District

One of the most powerful ways for children to achieve in mathematics is to experience constructive real-life mathematical situations in the classroom (Commission on Standards for School Mathematics, NCTM, 1987). Examples involve problems drawn from the child's local environment, for instance, actual and simulated excursions such as estimating gas mileage while riding in a car or estimating the amount of change to be received while shopping at a local store. Primary students might learn to identify coins by using real coins to purchase ice cream from the school cafeteria. Intermediate students can learn about percent by computing sales tax on actual purchases at a local department store or discounts on records or tapes from a music store. Situations like these, which will be referred to as "everyday mathematics," fill the child's world with activities involving thinking skills and estimation. Beginning in kindergarten, students should form the habit of estimating before calculating answers (Mathematics Framework for California Public Schools, K-12, 1985).

WHAT IF... I have $10.00 to spend on a music tape, how much tax will I pay? What will be the total cost? How much cheaper is the price in one store than another? What might be a good price? If I can get the tape for 25% off, what will I have to pay?

Mathematics involving such "what if" questions is the mathematics encountered in everyday situations. It is the mathematics one does in coping with the real world. Pencils and calculators are not always used, but thinking skills, approximation, estimation, and common sense are required.

Motivation for Learning

Students who apply skills in their daily lives are much more motivated to internalize the learning of these skills. For instance, students who count money to see if they have enough to purchase cookies are much more motivated than students who count plastic money for no real reason; students counting M&M's so that each student receives an equal amount are more highly motivated than students using plastic counters to learn counting. Representing these experiences on paper or on the chalkboard is more meaningful because students are involved in the action. Students using everyday mathematics are motivated to achieve beyond their grade-level objectives because their goal is to solve real problems.

A challenging activity might involve developing a booklet of everyday problems to be solved by the class. Such a publication can be extremely motivational when students share problems with family and friends. Ownership of problems motivates students, and this increased motivation leads to increased learning.

Emphasis on Interaction

Everyday mathematics can utilize common experiences which stimulate interaction between students and the teacher as well as among students. Since most classes have students who shop at the same stores and attend the same recreational activities, students can work in groups to formulate and solve everyday problems. "Interaction among children is one of the chief motivators for the learning process. Children's thinking and insight are stimulated by explaining their solution processes to others, as well as by the ideas and questions offered within the group. They come to examine a problem more objectively and begin to use new perspectives when they are exposed to different points of view" (Gilbert-MacMillan and Leitz, 1986). This interaction promotes problem-solving strategies and develops thinkers. The teacher acts as a catalyst to stimulate thinking by responding to the students' inquiries as they think through the development and solution of relevant local problems. Children are encouraged to offer varied approaches to a solution. An example of interaction that might occur follows:

Teacher: All 18 of you brought in your $1.00 for the pizza party. What shall we do?
Student 1: We can get pizza from the cafeteria for $3.00 for each pie.
Student 2: If each pizza can be cut into 8 pieces, we will need 4½ pizzas for each student to get 2 slices.
Teacher: We cannot buy ½ of a pizza.
Student 2: We will have to buy five pizzas. That will cost $15.00.
Student 3: Can we have soda?
Teacher: Six packs of Pepsi will cost $1.89.
Student 3: Then we will need about $6.00 for soda, but we only have $3.00 left.
Student 4: We can get two-liter bottles for $0.79 each. Six people could easily share a bottle.

Three bottles times about $0.80 each is $2.40.

Student 5: Mrs. Jones, do you like Pizza?

Teacher: Why do you ask?

Student 5: Well if you don't like pizza, then it would be easier to buy 6 pies, cut them into 6 slices each, and forget the soda. Then we won't have to fight over the extra slices and worry about giving money back.

This type of interaction contracts sharply with that which occurs when a teacher stands in front of the room with answer book in hand, while students recite their answers to the odd-numbered exercises.

Everyday Mathematics and Textbooks

Because many word problems in textbooks are contrived, it is critical to include locally-developed problems in mathematics lessons. Textbook problems often are so predictable in their makeup that students do not even bother to read them, but just extract the numerical data and compute (Paige, 1988). For example, adding prices from a menu in a local fast-food restaurant may be more meaningful than doing so from a menu in a textbook. Measuring distances that involve local landmarks can be more effective than computing distances in textbook problems. Students improve their understanding of measurement by measuring things meaningful to them. Recording and comparing growth patterns or performances in local "olympic games" are suggested activities. Problems involving personal statistics of students (e.g., height or number of siblings) take on added significance to them.

When students use only word problems from textbooks, there is little transfer to the everyday world. Since the students are consistently working toward the type of solution usually found in answer keys, they may not use higher-level thinking skills nor have the opportunity to answer the "what if" questions commonly encountered in everyday problems.

Everyday Mathematics and Its Integration with Language Arts

Everyday mathematics activities provide opportunities for students to develop oral and written communication skills. Students can discuss their daily experiences and mentally "hold conversations with themselves" to organize language that can be understood by others. Reading and writing skills improve when students are writing about topics they know well and have experienced. Furthermore, an interest in word problems increases when there is a positive response from peers and a sense of ownership. As an example, when students' mathematics problems are duplicated for class members, they are motivated not only to solve the problems but also to improve their language so that peers are able to read and understand their written work.

Conclusions

In today's world, estimation, mental arithmetic, and knowing what operation to use in solving problems are more important than the actual algorithmic method of calculation. However, in many classrooms, students spend most of their time on repeated manipulations. What is even more tragic is that, all too often, worksheets are simply marked as right or wrong, with the assumption that errors are mainly due to carelessness. Everyday mathematics can be highly diagnostic and offers teachers opportunities to detect incorrect methods employed by students and to improve students' problem-solving ability.

If one believes that understanding solutions and developing thinking skills are as important as getting correct answers, then everyday mathematics is a method to consider. Furthermore, if one truly believes that the primary function of school is to help students survive in the real world, then the real world should be the primary source of learning experiences. In this context, everyday mathematics becomes a vital component of the mathematics programs in every school.

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Reprinted from *1989 Yearbook* of the Pennsylvania Council of Teachers of Mathematics.

Problem Solving Ideas

Betty J.W. Morgan

You may not be familiar with the following strategies that are often used in problem solving. You can prepare a poster that includes these strategies and encourage your students to refer to the list as they work. (*Arithmetic Teacher* - December, 1985) The months that the strategies were discussed in the *Arithmetic Teacher* are in parentheses.

1. Draw a Picture (Oct. 1984)
2. Find a Pattern (Jan. 1985)
3. Guess and Check (Sept. 1984)
4. Logical Reasoning (Feb. 1985)
5. Organize a List (Dec. 1984)
6. Experimentation/Simulation (Nov. 1985)
7. Solve a Simpler Problem (Mar. 1985)
8. Work Backward (Apr. 1985)
9. Write an Equation (May 1985)
10. Make a Table (Nov. 1984)

I hope you find these valuable articles helpful to you and your students.

Reprinted from the South Carolina newsletter, *The Mathmate*.

Professor Bear anticipates the results of his new efforts to implement the Standards.

But Where Do I Get Problems?

Stephen Krulik

Teachers everywhere are in agreement that problem solving is one of the most important skills that must be taught in the schools if we are to prepare our students to survive in the world they will enter as adults. Although the National Council of Teachers of Mathematics stated back in 1980 that "Problem Solving must be the focus of school mathematics in the 1980's", it has not been until the last year or so that many classroom teachers began to fully comprehend what problem solving meant, and how it differed from what they had been teaching as "word problems".

It must be understood that this author does not consider the word exercises usually found in textbooks to be problems in the problem solving sense. That is, "word problems" as they are commonly called, are simply exercises designed to illustrate a particular concept, skill, or algorithm that has just been taught. And, if the student can recognize the particular algorithm involved and can *apply* it correctly, then that student will arrive at the correct answer. This is not true in problem solving. There is rarely a single algorithm that can be applied to a problem that will lead directly to an answer. *Problems* are situations which confront a person and for which an immediate path to the solution is not visible. The problem solver must synthesize a great deal of what he or she has already learned, and use this knowledge to find a solution. Careful thought is required, not merely recognition. Notice, too, that in problem solving, it is the *solution* (the method) that is just as important as the *answer*, if not more so. In some cases, the solution may actually be the answer!

If the "word problems" in textbooks are not really problems, how do teachers find problems to provide their students with practice in problem solving? Where do these problems lurk? It is the purpose of this paper to provide you with some readily available sources of problems for problem solving for your students.

1. *Modify the "problems" in your students' textbooks from single stage exercises to multi-stage problems.* Most of the exercises that appear in your students' textbooks are simple one-stage exercises designed to illustrate some skill. If you take these and modify them into two or more stage problems, then they become real problem solving activities for the students. For example, a traditional exercise in textbooks is often one such as this:

(original) If I spend $1.50 for soda, $4.25 for pizza, and 75¢ for ice cream, how much do I spend altogether?

To solve this exercise, all the student has to do is to add the three prices given. We can modify this into a two-stage problem as follows:

(modified) If I spend $1.50 for soda, $4.25 for pizza and 75¢ for ice cream, how much change will I receive from a $10 bill?

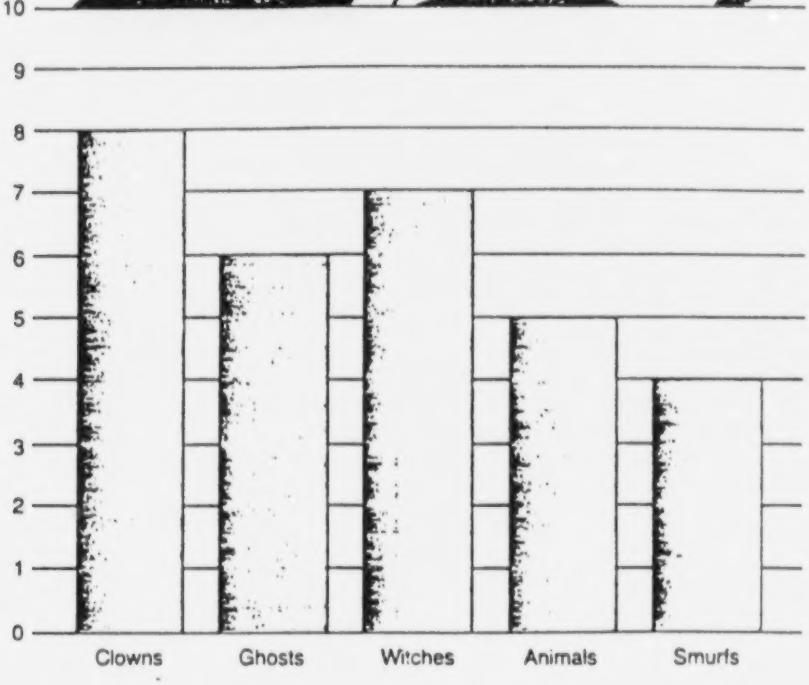
Or this one:

(original) There are 5 fish in a fish tank. Each fish eats 7 grams of fish food per week. How much fish food do all the fish eat in one week?

(modified) There are 5 fish in a fish tank. Each fish eats 7 grams of fish food per week. If I buy a can that holds 630 grams of fish food, how many weeks will it last?

2. *Use the centerfolds of well known magazines.* Many of the journals in our field such as *The Arithmetic Teacher* and *The Mathematics Teacher* place posters and other activities in the centerfold of each issue. These are usually excellent problem solving activities. For example, in the October 1983 issue of *The Arithmetic Teacher*, the poster focused on "Costumes at the Halloween Party". The centerfold provided a bar graph showing the number of students who came to the party dressed in various costumes, and then provided questions relating to the theme and the graph.

(continued on next page)



Costumes at the Halloween Party

7. How many more clowns than ghosts were there?
8. How many more clowns and ghosts than witches and animals were there?
9. How could the teacher divide the class into two equal groups without splitting up any students with similar costumes?
10. What fractional part of the class wore ghost costumes?

Notice that these centerfolds actually appear in color, are attractive, current, topical and interesting to the students. By building a file of these posters, you develop a ready source to be used at the appropriate time and season for years to come.

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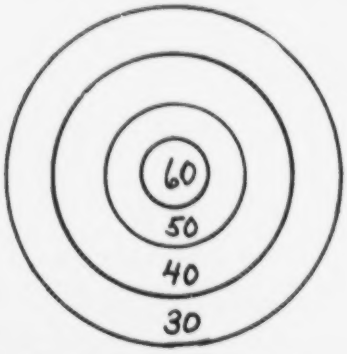
3. *Modify existing problems so that they cause students to utilize other strategies.*
   Many of the "word problems" that appear in the students' textbooks involve one single strategy or approach. Very often these can be modified to include other strategies as well. In addition, they can provide the students with the opportunity to examine problems that may have more than one single answer.

(original) Mary threw three darts and hit the 30, 50, and 60 on the target. How much was her total score?



This exercise requires the student to merely add the three numbers and get 140 as the answer. Let's modify it to make it into a problem involving "guess and test" as well as addition. Notice, too, that the problem now has more than one answer. This is often bothersome to students who are used to looking for a single answer. This in itself is a valuable experience for your students.

(modified) Mary hit the target with three darts and scored a total of 140. What might have been her three scores?

Or this one:

(original) How much does it cost for one hamburger, a portion of french fries and one medium sized Coke?

(modified) Juan spent $2.35 for lunch. What might he have eaten?

MENU

Hamburger	$ 1.25
Tuna Sandwich	$ 1.35
Hot Dog	$ .95
Cheese Sandwich	$ 1.05
French Fries (portion)	$ .75
Coke.....small	$ .55
medium	$ .65
large	$ .75

4. *Take your students "Back to Basics".*
   In the past few years, there has been a great clamor for the schools to go "Back to Basics". So, why not take your students back? Way, way back? Back to the 1800's for example. You can often find old textbooks at flea markets or garage sales. These books will often provide problems for your students which are quite interesting. Notice, too, that they provide the students with a taste of the culture of the time. Here are three sample problems that are taken from *The New Arithmetic*, edited by Seymour Eaton, and sold by D.C. Heath Company in 1895.
   (a) A farmer's wife sold a grocer 15 dozen eggs at 14 cents, and 27 pounds of butter at 22 cents; she received in payment 12 pounds of sugar at 11 cents and cash for the balance. How much cash did she receive?
   (b) A railway charges a cent a mile for the first fifty miles for carrying a cord of wood, and then 3 cents for every 4 miles beyond the fifty. What will it cost to cary 250 cords 90 miles?
   (c) A man bought on speculation 68 horses at $84 each; 11 of them die; at how much each must he sell the others to gain $444?

5. *Use commercial sources to build your problem collection.*
   There are many sources of problems available to the classroom teacher. Some of these are excellent; others are merely rehashes of standard word exercises dressed up in a fancy package. You will have to examine these sources individually and decide which of them, if any, are most suitable for your own situation. (The names of some of these are included in the bibliography at the end of this article). Purchase those that you feel will be of most value. Then share them with other teachers. In this way, your collection should grow.

6. *Use your students' own problems.*
   Turn the tables on your students. Ask *them* to provide problems for their classmates to solve. Then save the best of them for future use. Very often you will find some real gems:
   (a) My father is 44 years old. My dog is 8 years old. If my dog were human, she would be 56 years old. How old would my father be if he were a dog?
   (b) Fifty students went on a field trip. Twenty came back. How many were lost?
   (c) A car goes 4,000 miles in 20 minutes. How fast does it travel?

It is problems such as these that make the entire problem solving activity so interesting.

In general, then, there are many excellent sources of problems for problem solving just waiting for teachers to discover them. It merely becomes a process of becoming more and more conscious of their availability, and putting them into a single, collectible source that you can and will use!

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This article appeared in the Spring 1986 *New Jersey Mathematics Teacher.*

An accident occurred at the school picnic and softball game, resulting in

A flight of fancy and the light of insight!

Field Trips: Avenues for Problem Solving

David Shipp and Charles R. Neatrour

Field trips provide excellent sources of learning experiences in social studies, science, and language arts. They can also provide excellent opportunities for mathematical problem solving. In fact, all four of these curricular areas can involve data collection, data recording, arranging data in tables, preparing graphs, and interpreting the data in a variety of forms. All of these and more are encompassed within mathematical problem solving.

Mathematical problem solving, as viewed by the authors, encompasses both verbal and written opportunities. However, our focus is on the use of manipulatives and involvement in the world around us. Surely the use of environmental settings selected for field trips provide such settings.

Recently, when David began to plan for his fourth grade class a field trip to visit historic Monticello near Charlottesville, he incorporated a number of problem-solving opportunities in several curricular areas. Many other teachers have taken this same field trip including the guided tour of the grounds, buildings and the gift shop; their students have learned about the historical significance of Monticello, about the impact that Thomas Jefferson had on Virginia and on U.S. history, about his scientific inventions, and about his broad range of interests. However, during these tours, how many teachers have taken the opportunity to incorporate problem solving and bring an application of mathematics into the day's learning experiences?

When you plan that next field trip, you can involve the children in the planning. You may be surprised by their ideas for numerous problems and for ways to seek data to answer a wide assortment of mathematical and non-mathematical problems. Your students can work in groups, selecting their own recorder, planning to collect data on some of the problems you have listed, as well as some that they have proposed. The number experiences need not be restricted to the historical sites, but can include the full day's activities from start to finish. Here are some suggestions:

1. How much will the field trip cost? You can provide the cost per mile to schedule the bus, the cost per hour for the driver if not included in the mileage fee, the admission price, and the lunch fee if packed by the cafeteria or purchased along the way. The students can determine the total cost as well as the cost per student.

2. How many pictures were taken if each student took four snapshots? It is also possible to collect these data on the bus on the way back to school and then determine the average number per student.

3. How old was Jefferson when he died? He was born in 1743 and died in 1826. How many years ago did he die? How old would he be today if he were still alive?

4. How many years have passed since the Declaration of Independence was signed on July 4, 1776.

5. What is the approximate distance from Monticello to Washington, D.C.? How many hours would it take for Thomas Jefferson to go from the White House to his home in Virginia if his carriage traveled eight miles in one hour?

6. How many hours each day is Monticello open if tours operate from 9:00 a.m. to 5:00 p.m.? Per week, if it is open only six days each week? For the month of February?

7. How many one-hour tours can there be in a single day if Monticello is open from 9:00 a.m. to 5:00 p.m., there are two guides, and each guide is allowed an hour for lunch?

8. A student paid $1.00 for a quill pen and $2.95 for a packet containing copies of the Declaration of Independence, Constitution and Bill of Rights. If he gave the clerk a $10.00 bill, how much change would he receive?

9. Upon return to the classroom, it was discovered that students had purchased 12 quill pens (@ $1.00), 8 gyroscopes (@ $3.95), 5 models of Monticello (@ $5.00) and 16 arrowheads (@ $.75). In addition to determining the amount of money spent in the tourist shop, the students can prepare a circle or bar graph reflecting the information.

A field trip like the one to Monticello can be an excellent opportunity for active learning and problem-solving experiences. Obviously, when students are involved in collecting data, solving problems, and writing their own problems, they learn a great many facts and ideas spanning several curricular areas—information that they will be able to access later. This active involvement in planning, collecting data and solving problems in several curricular areas will frequently motivate students to pursue learning activities that extend beyond the school's classroom.

Charles is on the faculty and James is a Graduate assistant at James Madison University.

31 Activities for the Problem Solving Standard

1. Given a picture, two children construct the object shown using blocks. (Shari Wilcox)

2. If the two of you have $1.00 and you spend 60 cents, what are some of the combinations of coins you can receive as change? (Laura Hotaling)

3. If the two of you have 50 cents in your coin bank, what are some of the coin combinations you might find? (Pauline Payette)

4. Have a box or basket of items. Have a pair of children sort them into two piles any way they like. After they describe their first sorting, challenge them to come up with several other sortings. (Deena Loan)

5. If the two of you have 12 pieces of candy and you have four kinds (orange, cherry, lime and grape), how many can you have of each kind? How many combinations can you find? (Lori Costello)

6. Create a mock grocery store on a table. Children work in pairs to decide what they will buy for a given amount such as $2.00. Have things children *want* so that choices will be discussed. (Christine Whitley)

7. Place 3 marbles or other items of one color and 3 of another color in a sack or can. Have two children make 20 or 25 draws, returning the item drawn following the recording of the draw each time. After several pairs have completed this task, let them share their results. This will help build the concept of probability. (Kelly Hildreth)

8. Have Othello game pieces of black and white. Have students predict how many blacks and whites they will get from 20 flips. Then have them conduct and record 20 flips. After several pairs have done this, they can present and discuss their results. Coins can also be used. (Angie Buczek)

9. Ten people are coming to dinner, you plan to seat them 2, 4, 2, 4 around your table. Three people cancel because they are ill. Will you and your partner suggest some ways to rearrange the seating? (Kathryn Colvig)

10. Give students four or five number sentences. Ask each pair to make up a story and solve the problem. (Kristin Givens)

11. Give children five numbers. Ask them to work in pairs to see how many ways they can product a sixth given number by operating on all or some of the given numbers. (Kristin Givens)

12. Give sums and provide manipulatives. Ask pairs to work together to find the sums any way they can. (Jeanie Mokwa)

13. There are 23 students in the class. At least 10 are boys and at least 8 are girls. Work with your partner to see how many boy/girl combinations you can find. (Stephanie Siebeking)

14. Ask the children to work in pairs to decide how they can fetch exactly 20 quarts of water with a 29 quart pail and a 3 quart pail. (Michele Chafee)

15. Label 12 pieces of paper 1-12. Draw them out in pairs and write the sums on the board. The children work together in pairs to decide what the six pairs of addends were. (Michele Chafee)

16. Make a 4x4 dot pattern on the board. Ask pairs of students to see how many shapes they can make with an area of 6. (Donna Davis)

17. John went to the store with $2.00. He saw candy bars 3/$1.00, gum for 30 cents, and fire balls 4 cents each. If he buys at most 15 items, what might he buy? Can you think of other combinations? (Brenda Sloss)

18. (Kindergarten) Each child holds a shape. Music plays. When the music stops, each child finds another child with the same shape. On alternate rounds, each child finds a friend with a *different* shape and trades. (Mary Elisa Boyer)

19. Have a box with red, blue and yellow chips (red = 1, blue = 5, yellow = 10). Ask pairs to determine what point values are possible if they draw 3 (or some other number of) chips. (Michelle Walmsley)

20. After a field trip, ask pairs of students to make up some story problems which reflect the experience. (Michelle Walmsley)

21. We have five books to place on the bookshelf. How many ways can we arrange them? (Kay Dillon)

22. You have one quarter, two dimes, three nickels and two pennies. How many different amounts can you make with 2 coins? 3 coins? (Kay Dillon)

23. Have children work in pairs to determine how much carpeting would be needed to carpet an area. This experience can include the fact that carpeting comes in rolls of a given width and given cost/yard. They can calculate waste, choose a color and pad, etc. (Mary Elisa Boyer)

24. There are three teams in the holiday games. Each team must play the others once. How many games will there be? What if there were 4 teams? 5 teams? 20 teams? (Eileen Tully)

25. Given some fraction stories with addition and subtraction required, pairs of children will use available manipulatives or some other strategy they devise to answer the questions. (This would be an *introductory* activity.) (Jeanie Mokwa)

26. Two children choose 9 consecutive numbers. They decide how to put them in a 3x3 grid pattern so that they get the same sum in all 8 vertical, horizontal and diagonal triplets. (Margaret Tilelli)

27. Have the students work in groups of 3 or 4 to decide a polling question (favorite TV show, distance from home to school, etc.), collect data, display it graphically, and write a set of questions. The whole thing can be used as a bulletin board display. (Laura West)

28. Have pairs of students predict the probability of drawing a club from a deck of cards. Then have them make 40 or 50 draws and graph their results. (Angela Buczek)

29. Parent's Night is coming. If we make art displays on 11"x14" paper, how many can we display along the front wall? Work in pairs. What if we turn them? (Christine Whitley)

30. Which will give you the most money deposited in your account, 25 cents/weekday all year, or $3.00/month during the school year? (Carolyn Croy)

31. Figure this one out in groups of 2 or 3. "There are two women. Each woman has two bags. In each bag there are 2 cats. Each cat has two kittens. How many women, bags, cats and kittens are there?" What if there were 3 of each? 4? 10? (Ame Harrell)

32. Work in pairs. There are 2 beetles in a box. If they triple in number every two days, how many will there be in 8 days? What if there are 3 beetles? 4 beetles? 10 beetles? (Pauline Payette)

33. Plan a party for the class. Work with your partner. Keep the cost under $25.00. Use the catalogue to determine prices. I also have a price list from the local Party Store.

These ideas are from Tech students in the K-8 certification program.

STANDARD 2:
MATHEMATICS AS COMMUNICATION

In grades K–4, the study of mathematics should include numerous opportunities for communication so that students can—

- ***relate physical materials, pictures, and diagrams to mathematical ideas;***
- ***reflect on and clarify their thinking about mathematical ideas and situations;***
- ***relate their everyday language to mathematical language and symbols;***
- ***realize that representing, discussing, reading, writing, and listening to mathematics are a vital part of learning and using mathematics.***

Focus

Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively. Communication plays an important role in helping children construct links between their informal, intuitive notions and the abstract language and symbolism of mathematics; it also plays a key role in helping children make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas. When children see that one representation, such as an equation, can describe many situations, they begin to understand the power of mathematics; when they realize that some ways of representing a problem are more helpful than others, they begin to understand the flexibility and usefulness of mathematics.

Young children learn language through verbal communication; it is important, therefore, to provide opportunities for them to "talk mathematics." Interacting with classmates helps children construct knowledge, learn other ways to think about ideas, and clarify their own thinking. Writing about mathematics, such as describing how a problem was solved, also helps students clarify their thinking and develop deeper understanding. Reading children's literature about mathematics, and eventually text material, also is an important aspect of communication that needs more emphasis in the K–4 curriculum.

This standard highlights the need to involve children in actively doing mathematics. Exploring, investigating, describing, and explaining mathematical ideas promote communication. Teachers facilitate this process when they pose probing questions and invite children to explain their thinking. Teachers also can assess students' knowledge and insight by listening and observing. The idea that children should learn mathematics meaningfully is implicit in this discussion, for meaningful learning is necessary if mathematics is to make sense and if communication is to be possible.

The Everyday World of Elementary Mathematics

Gladys B. Hamilton

Teachers need to rethink their use of terminology as they teach so that they will afford the children in their classes with an extensive vocabulary of the language of mathematics. Check yourself each time you go to teach a lesson so that the children in your class speak MATHEMATICS properly.

A "New" Language

The very early years is the time to teach mathematical language. Give them the correct term and what it means. At this time the children are fascinated by the grown-up words and will try to remember them and what they mean, so that they can surprise adults by spouting off the words as they describe what they are doing at school. How many parents ask that old question, "What did you do in school today?" If the child has learned a "big word" he is usually anxious to surprise the parents, so he uses it as he tells about his adventures in class.

In a first grade that I taught many years ago, I had a vast variety of children. In that class children's ability ranged from genius level to immature individuals who didn't even know how to write their names. Yet all of them learned to *speak* "MATHEMATICS." I told them that as the year went along, I was going to teach them about this new language which none of them spoke.

Words Cause Difficulties

When previewing the scores on several standardized tests which had been given to the students, I noticed that many of the children missed the same items. Now that is usually true when we check results on any test and we then must investigate why these same items are answered incorrectly.

In my investigation, I discovered that many of the mistakes were similar in nature and that the question was worded so that correct mathematical terms were used.

In interviewing some of the students that I knew were rather good math students, there was a commonality to their answers and the reasons for their decisions. As many of us are aware, most standardized tests are so constructed that the answers are multiple choice for easy scoring and correcting. Unless we ask children *how* they decided upon their answers, we would never find out!

I chose items with specific mathematical wording which I thought was one of the reasons that the children had missed some very simple problems. I made samples similar to these and then interviewed the students individually as they solved the problems. Their comments were most interesting. I will give some instances where children's errors were caused by not understanding the language.

One of the samples on a third grade test was:

Tom had 44 balloons to sell.
He broke 26 balloons when he fell.
What was the remainder he had to sell?

Some of the children gave me the right answer without knowing what it really asked. They subtracted because "There's a big number first and a small number. You always subtract when it says that. "Or, "He fell and broke some, so you subtract."

When I asked them what a *remainder* was, the couldn't tell me.

The same type of answers were given when the question was "What is the difference?" However, on this one, I got some new replies. "I didn't answer that one 'cause the answer was not there." Or, "I marked *not here* because it didn't have the difference written, just numbers."

The idea of *difference* is another story. When children think about "difference" they think of the concept of "like and unlike." Therefore, when they are asked to find the difference, they think of what is unlike in the problem. So they tend to answer that one number is more than the other, or that in one they had blue balloons while in the other part they had yellow balloons. Of course, on the answer sheet only numbers are listed and so the children feel that this is something which has not been covered in the math class.

Using Synonyms

Using a variety of terms for the same word is also useful to the students. They need to know that there are many names for the same idea. An example of the concept of many terms for the same is when a child is asked to pick out the polygons shown. Usually they will pick out only the figures for which they have not learned specific names. In one instance when I tested one child, he said there were none. He knew the names for each figure such as circle, square, rectangle, hexagon, etc. He did not know that these could be called by other names.

In working with children who have learning problems and testing them, I found that many errors were made because one word and one word only had been stressed with their learning of a skill. James, an average child in fourth grade, could not find out how many carrots were in a bunch. As he read the sentences to me, it soon became evident that he didn't know what a bunch was, a group or the total. However, if the question asked for, "How many in a set?" he could immediately answer. Once again errors were made through lack of understanding of the terminology and an insufficient knowledge base of synonyms.

Math Language

Somewhere along the line some teachers must have forgotten to speak and teach "Math Language." The primary level teachers should start to teach the correct terminology of fractions with explanations about fractions. Then the intermediate teachers could continue and reinforce proper terminology. The language of mathematics would become part of children's active vocabulary.

Children like the terms "numerator" and

"denominator" if they are related to them with some information about their names and meanings. Put the definition into terms that the child of a certain ability level will understand.

As I sat in a workshop for teachers, right before school started, I was surprised to hear a middle school math teacher complain about fractions. She was lamenting the fact that many of her students did not know the terms most commonly used with fractions. She stated the case of some students who wanted to know what she meant when she stated that she wanted them to divide the numerator by the denominator. "Which one is the numerator? Which one is the denominator?" They hadn't even been able to correctly repeat the words. "Is it the top one or the bottom one?" was commonly heard. Finally after much tribulation she got through to them which was which. However, she felt that this was only a temporary victory.

Entertaining with Mathematics

One night near the end of the school year, my class always gives a parents' tea. The children help to decide on the entertainment of the evening. The one year the students decided to demonstrate what they had learned in mathematics for the year.

After the decision is made the students must think of ways of carrying out their ideas. Soon it was decided that each child would describe one part of mathematics to the parents. So each chose an area in which he/she felt comfortable and we worked from there.

Soon the children were agog with excitement. They would chatter daily as they discussed what they were going to say. It was surprising to hear them as they explained what the process for addition, subtraction, multiplication or skip counting was. They used so many of the "big words" and then described the meaning of these words as if the parents wouldn't understand unless the words were explained to them.

The night of the tea finally arrived. As a group of twenty-five students eagerly awaited backstage for it to be time to start, the chatter grew as each one was sure he/she would forget one of his words. Anxiety was at top level by the time we were to start.

As each child took his/her place on stage and reported on one area of math for the year, the audience became more quiet during the explanation and more wild with their applause as the child completed his/her part.

At the close of the entertainment and other awards, the children served punch and cookies which they had made as part of their measurement unit. They were so proud of themselves, but the parents were even prouder as they talked back and forth. Some of the comments which were told to me or that I heard were:

"I didn't realize that John knew what all those words meant that he spouted at home."

"This is the surprise of the century for me. Susan hated math in first grade."

"Joey is not too quick to work out the answers on his work but I am glad to see that he really knows the ideas."

"Paulo made a good adjustment when he came to your room for he understood everything by the way you communicated with him. You used precise language." (This was from a family who had moved here from Brazil to work as scientists for Corning Glass Works.)

Concluding Remarks

Yes, the language of mathematics is a precise one with many "big words" but if time is taken to explain the words as they are used, the work involved is also explained more clearly and slowly so the child tends to retain more of both the LANGUAGE and the PROCESS.

Reprinted from the *New York State Mathematics Teachers' Journal.*

Take the plunge - get into the swim---

Stay in the mainstream with the new NCTM Standards!

21 Activities for the Communications Standard

1. Have 3 or 4 children work together in a learning center with blocks. They can decide on something to build such as a farm or a neighborhood. (Shari Wilcox)

2. Have children work in groups of 2 or 3 to solve problems like those listed and described in the Problem Solving Standard. After groups have had time to work, have a sharing time during which children explain their solution strategies. (Pauline Payette)

3. After solving problems, children can describe their solution strategies in writing or orally. (Marnea Prestel)

4. Have children interview each other to collect numerical data, perhaps one piece of information for each pair of people (e.g., class of 20 has 10 questions. After the interviews, each pair gathers the data from other teams for one question and makes a chart or graph. (Christine Whitley)

5. In number 4 above, each pair could write a report on the answers to their whole set of questions.

6. Construct 3 or 4 or 5 shapes such as equilateral triangle, isosceles triangle, square, rectangle, circle. Use some of them to make a shape. Trace *only the border*. Ask pairs of children to see if they can find one or more ways to solve your puzzle. (Marnea Prestel)

7. Children can write short stories with numbers in them. For example, you can write "3" on the chalkboard or "6x3 = 18" "3 + 4 = 7" and they can write a story which is about three or which is based on the number sentence. After talking about taking a trip, have the children write a story which includes departure and arrival times, distance, cost of lunch and other numerical information. (Shari Wilcox)

8. Each child solves a problem independently; then pairs of children get together and discuss their solution strategies. Each child then describes *in writing* how his or her partner solved the problem. Then the partner adds comments at the end of agreement or disagreement. (Margaret Tilelli)

9. After the teacher has made a presentation, pairs of children work a sample exercise together to see if they are following okay. (Laura West)

10. Tell a simple story such as "On his way to school, John saw two cats," etc. Write "2 cats," etc. on the chalkboard. Ask children to see how many number sentences and corresponding written statements they can write. For example, "John saw 2 cats and 4 birds. 2 + 4 = 6." (Brenda Sloss)

11. Lead a discussion of number ideas at "calendar time" each morning. (Deena Loan)

12. Write a numeral (e.g., 4) on the board. Pass out pictures. Have two children work together to circle groups of four items in the pictures.

13. When introducing a new topic, have children write about an experience in which they have encountered this topic or concept. What were they doing? How did they feel? (Carolyn Croy)

14. Have a treasure hunt. One partner is blindfolded; one is not. One reads the map and gives only verbal directions and clues to his or her partner to find the treasure. (Kelly Hildreth)

15. Any mathematical tasks in which children work with partners and which include sharing of results and strategies involve communications.

16. Write a paragraph or two describing how addition is used in your home. (Jeanie Mokwa)

17. Ask pairs of students to see how many fractions they can name which are bigger than ¼ and smaller than ½. Have sharing afterwards of strategies and solutions. (Michele Chafee)

18. Write some problems on the board. Have the children work in pairs to (1) make up a story, and (2) solve each one. (Melissa Baker)

19. What is 32? Pairs of children can create art, write a story, draw a picture, etc. They can share their ideas. (Laura Hotaling)

20. Pairs of children work together to see how many Cuisenaire Rod combinations of 2 rods they can get for 10 (orange) and other colors. They write addition sentences. This can be repeated for 3 rod combinations, too. (Kay Dillon)

21. Give each pair or group of three a mathematical number sentence or a couple shapes. They will prepare and act out a little skit. (Mary Elisa Boyer)

These activities were developed by students in the Tech K-8 certification program.

STANDARD 3:
MATHEMATICS AS REASONING

In grades K–4, the study of mathematics should emphasize reasoning so that students can—

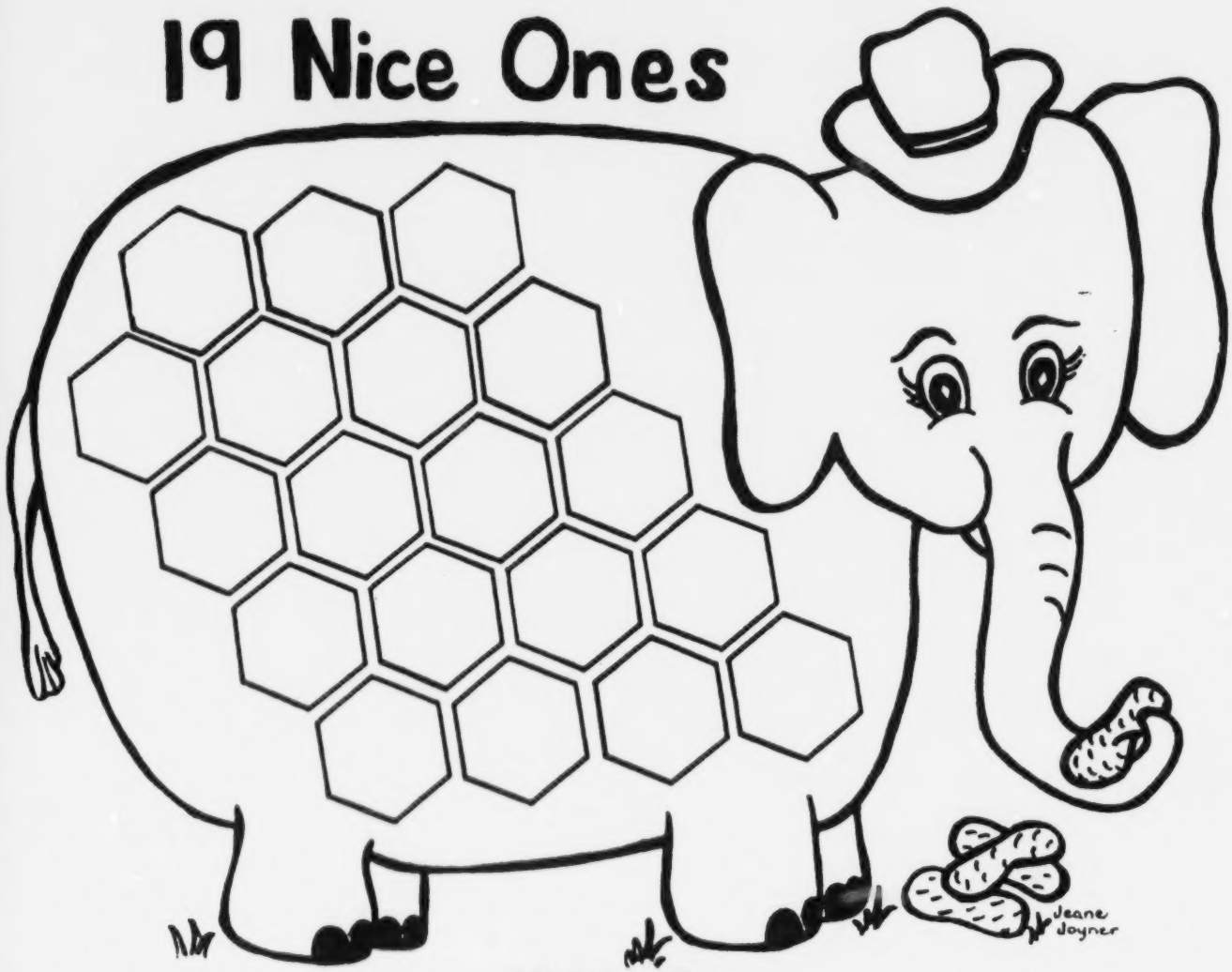
- ***draw logical conclusions about mathematics;***
- ***use models, known facts, properties, and relationships to explain their thinking;***
- ***justify their answers and solution processes;***
- ***use patterns and relationships to analyze mathematical situations;***
- ***believe that mathematics makes sense.***

Focus

A major goal of mathematics instruction is to help children develop the belief that they have the power to do mathematics and that they have control over their own success or failure. This autonomy develops as children gain confidence in their ability to reason and justify their thinking. It grows as children learn that mathematics is not simply memorizing rules and procedures but that mathematics makes sense, is logical, and is enjoyable. A classroom that values reasoning also values communicating and problem solving, all of which are components of the broad goals of the entire elementary school curriculum.

A climate should be established in the classroom that places critical thinking at the heart of instruction. Both teachers' and children's statements should be open to question, reaction, and elaboration from others in the classroom. Such a climate depends on all members of the class expressing genuine respect and support for one another's ideas. Children need to know that being able to explain and justify their thinking is important and that how a problem is solved is as important as its answer. This mind-set is established when children have opportunities to apply their reasoning skills and when justifying one's thinking is an expected component of problem discussions.

19 Nice Ones



19 Nice Ones

by Jeane Joyner

Topic:	1 to 1 correspondence, patterns, strategic thinking
Grade Level:	K - 6
Time:	10 - 15 minutes
Number of players:	2
Materials Needed:	Gameboard (see figure above) 19 markers
Object:	To win a player must force his opponent to take the last marker.
Rules:	1. Cover all the spaces with the markers. 2. In turn players may take 1, 2 or 3 objects. 3. Player loses who takes the last marker. 4. The champion is the winner of 3 out of 5 games.

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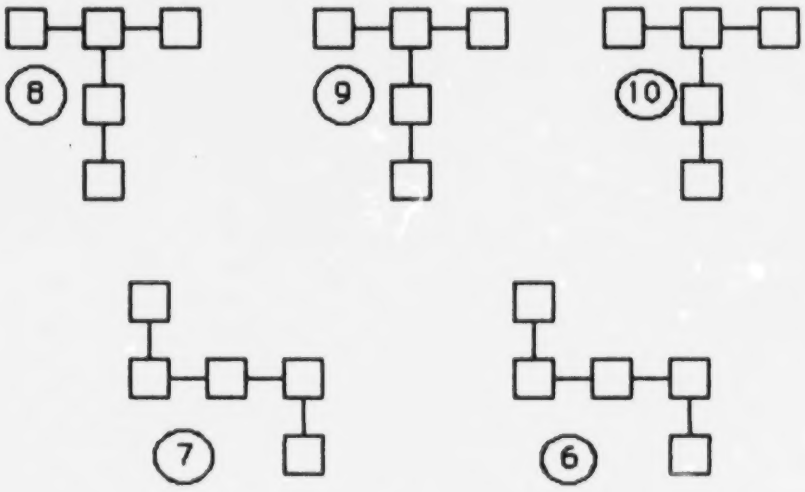
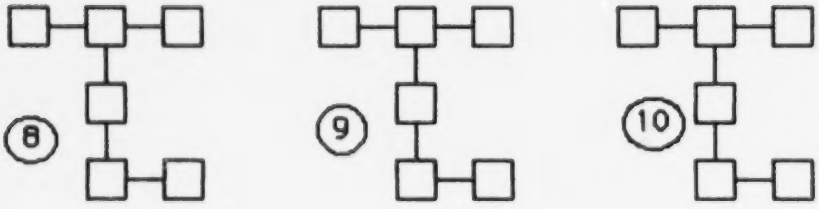
PROBLEM SUMS

Put 1, 2, 3, 4, and 5 in the boxes. The sum across and the sum down must equal the number in the circle.



For these use 1, 2, 3, 4, 5, and 6.



Now invent some more.

Which number combinations can be used? Are there any restrictions?

20 Activities for the Reasoning Standard

1. Play a card game with shapes. Three children play. One draws a card with a shape and describes the shape. The other two try to guess the shape. (Shari Wilcox)

2. (Spatial reasoning) Given two-dimensional grid-type drawings, have children work in pairs to decide which ones will form a cube when cut out and folded. (Pauline Payette)

3. (Information not needed) If there are 26 boxes and 12 roses will fit in each box, how many boxes will be used to pack 64 roses? Will the last box be full? (Karen Fallin)

4. (Similarities and Differences) Give each pair of children a piece of paper with a triangle, parallelogram, rectangle and circle. Ask them how they are alike and different. After a few minutes have a class discussion, listing similarities and difference *in their words*. (Ame Harrell)

5. Construct 3 or 4 or 5 shapes such as equilateral triangle, isosceles triangle, square, rectangle, circle. Use some of them to make a shape. Trace *only the border*. Ask pairs of children to see if they can find one or more ways to solve your puzzle. (Marnea Prestel).

6. Have pairs of learners use play money to determine the various combinations of 4 or 5 or 6 coins which can be used to make a certain value such as 27 cents. (Christine Whitley)

7. Farmer Brown has a total of seven cows, pigs and chickens. How many of each might he have? How many combinations can the two of you find?

8. (Spatial) Have children visualize how many triangles of a given size or shape will fit inside a given figure. (Margaret Tilelli)

9. (Trial and Error) Give pairs of children some shapes. Have them explore different ways they can put these shapes together to make other shapes. They might trace these shapes and let them serve as puzzles for others in the class (homemade tangrams!). (Stephanie Siebeking)

10. In the learning center, have some cards with shapes on one side and descriptions on the other. The cards can be used two different ways: (1) given shapes, students can describe them, and (2) given descriptions, students can name the shapes. This makes a good partner activity. (Laura West)

11. Number game to be worked in pairs. I am a (2 or 3) digit number. I am (odd or even). The sum of my digits is ( ). Is there another possibility? (Elyse Williams)

12. Put several objects in a bag. Have children work together taking turns reaching into the bag, handling an object without looking and trying to guess what it is. (Deena Loan)

13. (Tower of Hanoi). Have a row of three pegs and a series of disks of seriated size. Place on one peg already seriated. Challenge pairs of children to move the stack to another peg while never placing a larger on top of a smaller moving only one disk at a time. Begin with 2; then see if you can do 3, 4 and 5.

14. Extending sequences:
    A. 2, 6,__,__, 10, 12,__,
    B. 0, 3, 6, 9, . . .
    C. Geometric patterns
    (Kelly Hildreth)

15. Have children make up sequences for others to figure out.

16. Give a partial sequence which is sufficiently open that several possibilities emerge, e.g., create two patterns for this 1,___, 5,___,___,___. (Angie Buczek)

17. Fold a piece of paper lengthwise and horizontally. While it is folded, cut a piece off. Ask students to draw a picture of what the sheet will look like when opened. (Kathryn Colvig)

18. After watching the metamorphosis of a butterfly, give pairs of students a set of cards with different stages on them. The children place them in chronological order and write a description. (Michele Chafee)

19. Here are two right triangles which are identical. See how many shapes you can make with them. Trace your shapes onto a piece of paper. Discuss results. (Donna Davis)

20. Use mathematical riddles. I am one less than 8 + 7. Who am I? (Melissa Baker)

These activities were developed by Tech students in the K-8 certification program.

STANDARD 4:
MATHEMATICAL CONNECTIONS

In grades K–4, the study of mathematics should include opportunities to make connections so that students can—

- ***link conceptual and procedural knowledge;***
- ***relate various representations of concepts or procedures to one another;***
- ***recognize relationships among different topics in mathematics;***
- ***use mathematics in other curriculum areas;***
- ***use mathematics in their daily lives.***

Focus

This standard's purpose is to help children see how mathematical ideas are related. The mathematics curriculum is generally viewed as consisting of several discrete strands. As a result, computation, geometry, measurement, and problem solving tend to be taught in isolation. It is important that children connect ideas both among and within areas of mathematics. Without such connections, children must learn and remember too many isolated concepts and skills rather than recognizing general principles relevant to several areas. When mathematical ideas are also connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics.

A classroom in which making connections is emphasized exhibits several notable characteristics. Ideas flow naturally from one lesson to another, rather than each lesson being restricted to a narrow objective. Lessons frequently extend over several days so that connections can be explored, discussed, and generalized. Once introduced, a topic is used throughout the mathematics program. Teachers seize opportunities that arise from classroom situations to relate different areas and uses of mathematics. Children are asked to compare and contrast concepts and procedures. They are helped to construct bridges between the concrete and the abstract and between different ways of representing a problem or concept. Learning and using mathematics are important aspects of the entire school curriculum.

Reflections on Mathematical Connections

Liz Hiller

In grades K-4, the study of mathematics include many opportunities to make connections so that students can: (1) link conceptual and procedural knowledge, (2) relate various representations of concepts or procedures to one another, (3) recognize relationships among different topics in mathematics, (4) use mathematics in other curriculum areas, and (5) use mathematics in their daily lives.

The integration of mathematics during the school day is vital in developing the understanding that math is not an isolated subject found only in the classroom. There are often many opportunities each day when you can use mathematics in other subjects. In Social Studies, when studying new countries, comparisons can be made as to their population, size, and proportion of land to water. During science, students can be expected to measure accurately, plot information on graphs, and design charts which accurately display data. The writing and sharing of ideas in math class can be urged. Through describing and articulating the processes used to solve problems the students will develop better understanding.

Inservice training and school workshops can be used to share ideas on how to implement the Standards. You might lead these activities and share with others. Topics may include the use of manipulatives to teach two-digit addition, how to use the calculator to teach math and the integration of writing in the math program.

You can design lessons around a variety of learning activities. You can develop students' mathematical understandings through the use of stories, manipulatives, pictures, and symbols. Mathematics is applied to the real-world through the use of meaningful stories. You can create stories that require the use of the particular concept or skill being studied. Given such stories the students can use manipulatives to determine the answers. In addition, you might reverse the order by creating a problem with the manipulatives and asking your students to make up stories that fit the set ups. As the students develop understanding semi-concrete activities are included. Pictures are drawn representing particular objects and again the student makes up a story that fits the pictures. Additionally, you can create stories and ask the students to draw pictures to find the solutions. As the student gains confidence and understanding the abstract use of symbols is added. The student is then able to understand various representations of a single concept. You can also have the students design problems and choose the tools they wish to use to solve the problems. Students can then work together in groups to find the solutions. Each group can share its solution with the class, creating an awareness of alternative solutions.

You can also develop activities which emphasize the development of relationships within topics. For example, when drawing triangles, students can be asked to measure the sides and add the sides to determine the perimeters. Students can develop charts and graphs to help them find solutions to puzzling problems.

Liz teaches in Arlington County.

CLUBHOUSE
THE GANG
KEP OUT
Have you heard? The New Standards have come in!
MATH TOMORROW WILL BE GREAT! SURE HOPE I CAN GET TO SLEEP TONIGHT
JN BOYD

Good News Travels Fast

19 Activities for the Connections Standard

1. Cooking projects combine kitchen experiences with number. (Shari Wilcox)

2. Scatter a total of 200 toothpicks of four colors on the lawn or sidewalk. Give each pair of children a cup. All children pick up toothpicks for one minute. Each group then prepares a bar or picture graph by color of those collected. They can also write addition and subtraction sentences based on their numbers compared to the total number possible or compared to other teams. (Pauline Payette)

3. Have two children decide where they would like to go in their state or country or in the world. Have them decide a route with 2 or 3 different stops and determine the distance. Trace a map and show the routes and distances. (Ame Harrell)

4. Ask pairs of children to make scale drawings of the school, the playground, a learning center or individuals to make scale drawings of their homes or one room in their homes. (Marnea Prestel)

5. Have the children plant seeds. Each day or every other day have them measure the plants and record their heights. Have them graph this information. (Christine Whitley)

6. To introduce two-digit by one-digit multiplication, write three problems on the board (e.g., 12x3). Have children solve them in pairs. Then have a discussion of strategies.

7. Have pairs of children write down lists of items in their neighborhoods with given shapes. (Laura West)

8. Read charts and graphs in social studies materials. (Elyse Williams)

9. Make a simple map with school, park, home, store, post office. Ask pairs of students to find the shortest route to run errands and so on which involve 2 or 3 stops. If cars are used, you can add the element of time to this so that routes are good compromises of time and distance instead of simply fastest or shortest.

10. In our science lab, we learned how to use the balance beam. Choose various small objects from around the room and compare their weights. For each pair, write the number sentence which expresses a balanced beam by adding other small objects. (Example: Weight of Math book + weight of 6 blocks = weight of Science book). (Brenda Sloss)

11. (Math and Science) Have several objects which can be placed in water. Use a balance to see which weighs more. Then see which ones float and sink. This provides experiences in weight *and* density. (Deena Loan)

12. Choose 2 cities, your own and another 100-200 miles away. Tell the children to work in pairs to do the following: Prepare an itinerary to leave city #1 at 9:00 a.m. If you average 55 miles while in the car, what time will you arrive in city #2 if you allow for sightseeing, lunch, gas stops and so on? Prepare all of the information in a chart or graph or write a narrative. (Lori Costello)

13. Have 3rd or 4th graders make story problems with illustrations for 1st or 2nd graders. (Kelly Hildreth)

14. Construct graphs from science data. (Angie Buczek)

15. After solving a multi-step story problem, have student pairs write explanations of their solutions, (e.g., most of the problems under the Standards in this list or the Problem Solving list). (Angie Buczek)

16. Have pairs of children design a treehouse and determine what materials they need. (Kristin Givens)

17. Work with a partner. Make a list of ways subtraction is used at home. (Jeanie Mokwa)

18. (K or 1). Give the children square papers and have them color them their favorite color. Use the squares to construct a primitive bar graph. (Melissa Baker)

19. Have the children work in groups of three to measure each other and make a graph. Then make a graph for the whole class. (Mary Elisa Boyer)

STANDARD 5: ESTIMATION

In grades K–4, the curriculum should include estimation so students can—

- ***explore estimation strategies;***
- ***recognize when an estimate is appropriate;***
- ***determine the reasonableness of results;***
- ***apply estimation in working with quantities, measurement, computation, and problem solving.***

Focus

Estimation presents students with another dimension of mathematics; terms such as *about, near, closer to, between,* and *a little less than* illustrate that mathematics involves more than exactness. Estimation interacts with number sense and spatial sense to help children develop insights into concepts and procedures, flexibility in working with numbers and measurements, and an awareness of reasonable results. Estimation skills and understanding enhance the abilities of children to deal with everyday quantitative situations.

From children's earliest experiences with mathematics, estimation needs to be an ongoing part of their study of numbers, computation, and measurement. It is important that children learn a variety of methods of estimating, such as the front-end strategy for computation and the chunking procedure for measurement. They also need to develop reasoning, judgment, and decision-making skills in using estimation.

Instruction should emphasize the development of an estimation mind-set. Children should come to know what is meant by an estimate, when it is appropriate to estimate, and how close an estimate is required in a given situation. If children are encouraged to estimate, they will accept estimation as a legitimate part of mathematics.

HOW MANY?

OBJECTIVE:

Students will work cooperatively in groups to:

* estimate quantities of objects in various containers;
* use calculators to determine averages;
* predict how many containers would be needed to equal a million "things."

PREPARATION:

Fill about 8-10 equal sized containers with various objects. Suggestions include: unifix cubes; wooden cubes; buttons; small spools of thread; square tiles. Have calculators available. Divide class into groups of four.

PRESENTATION:

Teacher holds up one of the containers, e.g. unifix cubes, and poses the question, "How many unifix cubes do you estimate are in this container?" Accept responses. List five or six responses and calculate the average (rounded off). Ask how many students think there are more than this average and how many think there are less. "How can we find out?" After counting the cubes determine the difference between the average and the real amount. Discuss ways that we could use to improve our estimation ability. Pose the problem - how many containers of unifix cubes would we need to have a million cubes. How can we find out?

Explain that the students will be working in groups to estimate the number of objects in a variety of containers. Each group will work with any five they choose. (Suggestion: Keep all containers on one table. Students take a container to their group and then replace it and pick another.) Distribute record sheets and check to be sure students understand how to complete the activity.

****

This activity is suitable for middle school, but can be adapted to other grades as well.

Contributed by Judy Koenig, Palms Junior High School in Los Angeles.

HOW MANY?

LIST THE CONTENTS OF EACH CONTAINER

COMMITTEE MEMBERS	A.	B.	C.	D.	E.
1.	est.#	est.#	est.#	est.#	est.#
2.	est.#	est.#	est.#	est.#	est.#
3.	est.#	est.#	est.#	est.#	est.#
4.	est.#	est.#	est.#	est.#	est.#
Average					
Actual #					
Difference					

HOW BIG IS A MILLION?

1. Select a container. What does it contain?_____
2. How many containers would you need to have 1,000,000 of these objects?_____
3. Would that many containers fit in this room?
   How would you go about finding this out?

Discuss this with your group. Summarize your findings below.

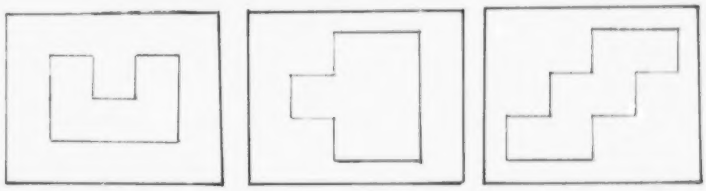
Contributed by Judy Koenig, Palms Junior High School in Los Angeles.

Kids Can Estimate!!!

by Jeanette Beer

Estimating is something we do almost every day as adults. We try to estimate how much steak we will need for dinner, or how much money we should withdraw for the shopping bill, or even how much time it will take us to run an errand. The list can be endless. Estimating is a concept which children are able to grasp, if they are provided with experiences to do so.

To introduce this concept, we played "cube guess" (**Linda Holden, K-3 Problem-Solving Games, Palo Alto, CA: Creative Publications, 1987**). This game requires players to choose a card and estimate the number of unifix cubes that will fit over the design, i.e.



After the player makes his/her estimation, he/she tests it out to see if it is correct. If the estimate is correct, the player keeps the unifix cubes and begins to build a train. After an allotted amount of time, the player with the largest train wins.

In addition to "cube guess" each child is given an opportunity to estimate each month. At the beginning of every month, I place a jar filled with something on the counter. For example, we might use candy corn in October, cotton balls in December, or jelly beans in April. Over the filled jar is the word **"ESTIMATE"**, and beside it is a small shoe box, a pencil, and a pad of paper. It is the child's responsibility to make an estimate before the end of the month during free time. At the end of the month, the children break up into cooperative groups and count a handful of the items. Then, as a class, we list the group totals on the board and add them up. The winner, each month, receives an **"Estimate Whiz Award of the Month."**

About the Author:

Jeanette Beer is a first grade teacher in the Farmington School System, Farmington, CT.

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Reprinted from the Spring 1989 issue of the Connecticut Mathematics Journal.

Developing Estimation Skills

Francis (Skip) Fennell

One of the most important goals within any mathematics program is to teach problem solving. Recent National Assessment (NAEP) results and recommendations by the National Council of Teachers of Mathematics have emphasized the need to carefully develop problem-solving skills at all levels of elementary education.

The ability to estimate is a crucial problem-solving skill. Estimation forces students to think about results, and note their reasonableness. The following activities emphasize the use of estimation in a variety of contexts.

Classroom Estimation Contests

Have the students estimate the following items. The students can enter their names and estimates for each contest.

Quantity - the number of small objects in a container (beans) or the number of large objects in a container (pencils).

Capacity - the volume of regular and irregularly shaped containers.

Weight - the weight of individual students, the combined weight of the class, or the weight of a variety of boxed materials.

Height - the height of individual students, the school, the classroom door, the playground fence, or even the flagpole.

Cost - the cost of various items found in a department store or food store.

Length - the total length of the playground or hallway.

You will be amazed at how close the estimations are once the students have an opportunity to practice these skills. Try an estimation contest once a week.

Price Estimation

Have the students estimate the cost of the objects listed on the chart show below. Ask them to find the actual costs in a newspaper or catalog, and to note these costs on the chart. Determine the difference between the estimate and actual cost.

Object	Estimate	Actual Cost	Difference
tennis racket			
television			
calculator			
box of cereal			
10-speed bicycle			
3-bedroom house			
soccer ball			
gallon of milk			

Time Estimation

Have the students estimate, then actually time themselves, the time needed to complete these activities:

walking or riding to school
writing the numbers from 1 to 1,000
jumping rope 30 times
running around the school building
writing a letter to a relative
writing the week's spelling words in a sentence

Estimating Project

Estimate the quantity and cost involved in serving pizza at school. What is the estimated number of slices to be served? What is the actual lunch count? What is the estimated total amount of money to be received for the pizza lunches? What is the actual amount?

This activity might be expanded to include estimating quantities and cost for a class Christmas lunch.

Estimation Discussion

Have the students discuss what they might purchase with the amounts of money listed below. Discuss suggested purchases and have the students bring in pictures or write about their proposed purchases.

$.10	$1,000
$1	$10,000
$10	$100,000
$100	$1,000,000

Estimate, Then Investigate

A number of common objects and occurrences that are great for estimation can be found everywhere. Use the "Estimate, Then Investigate" chart shown below for this activity. Make sure the students first estimate their answers, and then check their answers.

Item	Estimation	Investigation
The number of students in my school		
The number of "Smiths" in the phone book		
The weight of a soccer ball		
The number of spokes on a bicycle wheel		
The number of ice cubes in an ice cube tray		
The number of students in a full school bus		
The number of pages in my math book		
The number of doctors in town		

This article appeared in the Spring 1986 *New Jersey Mathematics Teacher*.

6 Activities for the Estimation Standard

1. Have two identical jars. In one place 50 jelly beans or shell macaronis. Fill the other jar to the top. Have the children work in pairs to estimate the total in the full jar. Then have them share their strategies and their estimates. (Pauline Payette)

2. Have pairs of children make estimates of the following and then have a discussion of their estimates and strategies:
   a. How many popsicle sticks will it take to cover this book?
   b. How many jelly beans are in this jar?
   c. How many people could stand in this room?
   d. How many square tiles like this would it take to cover our floor)

   (Christine Whitley)

3. Given one jar partially filled with 100 shell macaronis and four other jars which are full, children will work in pairs to estimate the number in the four jars. Then they will have a class discussion of strategies and results.

4. Two children work on this together. (Grades 3 and 4 without manipulatives or paper and pencil). If you and a friend go to the candy store with 75 cents, about how many four cent gum balls can you buy? Have children discuss strategies and solutions. (Elyse Williams)

5. Have 7 or 8 containers and 2 or 3 items which could be used to fill them (such as shell maraconi, styrofoam packing pieces). Have pairs estimate how many of each will be needed to fill each container, then count. Share strategies. (Jeanie Mokwa)

6. Have pairs of children walk off a given distance toe-to-heel as a starter. Then have each pair ESTIMATE the number of toe-to-heel measures in 4 or 5 other distances. After the pair finishes the estimates, they will actually walk off these distances. After all have finished, they will discuss their estimates and strategies. (Mary Elysa Boyer)

Old Prof. Bear must have been more exciting than usual when he went over the New Standards during the Spring term.
He must have been! I heard the whole class came out of hibernation two months early!
T.N. BOYD

Discussion of the new Standards is getting quite lively.

Appendix

How sad Little Dickie would become whenever Professor Bear mentioned long division and square roots... In fact, his classmates called him "Mopey Dick."...But now

There's been a whale of an improvement.

NCTM Position Statements

Calculators in the Mathematics Classroom

The National Council of Teachers of Mathematics recommends the integration of the calculator into the school mathematics program at all grade levels in classwork, homework, and evaluation. Although extensively used in society, calculators are used far less in schools, where they could free large amounts of the time that students currently spend practicing computation. The time gained should be spent helping students to understand mathematics, to develop reasoning and problem-solving strategies, and, in general, to use and apply mathematics.

At each grade level every student should be taught how and when to use the calculator. To use calculators effectively, students must be able to estimate and to judge the reasonableness of results. Consequently, an understanding of operations and a knowledge of basic facts are as important as ever. The evaluation of student understanding of mathematical concepts and their application, including standardized tests, should be designed to allow the use of the calculator.

The National Council of Teacher sof Mathematics recommends that all students use calculators to:

- concentrate on the problem-solving process rather than on the calculations associated with problems;
- gain access to mathematics beyond the students' level of computational skills;
- explore, develop, and reinforce concepts including estimation, computation, approximation, and properties;
- experiment with mathematical ideas and discover patterns;
- perform those tedious computations that arise when working with real data in problem-solving situations.

The National Council of Teachers of Mathematics recommends that publishers, authors, and test writers integrate the use of the calculator into their mathematics materials at all grade levels.

Mathematics Leaders in Elementary/Middle Schools

A key component in the improvement of mathematics instruction for all students is greater attention to the elementary/middle school mathematics program through the provision of stronger mathematics leadership at individual school.

School mathematics leaders are essential for raising the level of mathematical knowledge and pedagogical competence of the staff, for coordinating mathematics instructional effort within and between buildings, and for helping to assure the implementation of a comprehensive, high-quality program. In addition, school mathematics leaders will support the roles and efforts of teachers, supervisors, and administrators who are increasingly concerned with promoting excellence for their students.

For these reasons, the National Council of Teachers of Mathematics advocates that every elementary/middle school identify a school mathematics leader to provide ongoing leadership and assistance in planning, implementing, and evaluating a comprehensive school mathematics program. School mathematics leaders should be available as a resource to all staff in the areas of curriculum design, professional development, teaching methodology, classroom management, selection of materials, and student assessment.

More specifically, school mathematics leaders may perform or assist in any, or all, of the following tasks connected with the essential instructional functions listed below:

1. **Curriculum design**
   a. Provide leadership in the development of mathematics instructional programs to meet diverse student needs
   b. Coordinate the implementation of a sound instructional scope and sequence for mathematics
   c. Facilitate the review and revision of the curriculum
   d. Integrate mathematics with other content areas

2. **Curricular content**
   a. Relate the content of state/provincial and local curriculum guidelines to the individual school's program
   b. Present and interpret mathematics content to meet teachers' needs
   c. Promote the importance of mathematics
   d. Keep abreast of national, state/provincial, and local recommendations for updating and revising curricular content by participating in the activities of professional associations

3. **Methodology and materials**
   a. Acquaint teachers with successful and innovative strategies for classroom instruction through demonstration lessons and conferences
   b. Consult with teachers in selecting and implementing activities that improve motivation and attitude toward mathematics
   c. Review, and recommend for acquisition when appropriate, teaching materials, calculators, computer software, etc., and assist teachers in integrating them into the program

4. **Assessment**
   a. Participate in the review and selection of means of assessment
   b. Assist staff in designing and using classroom tests
   c. Assist staff in interpreting and using test data and placing students in appropriate activities

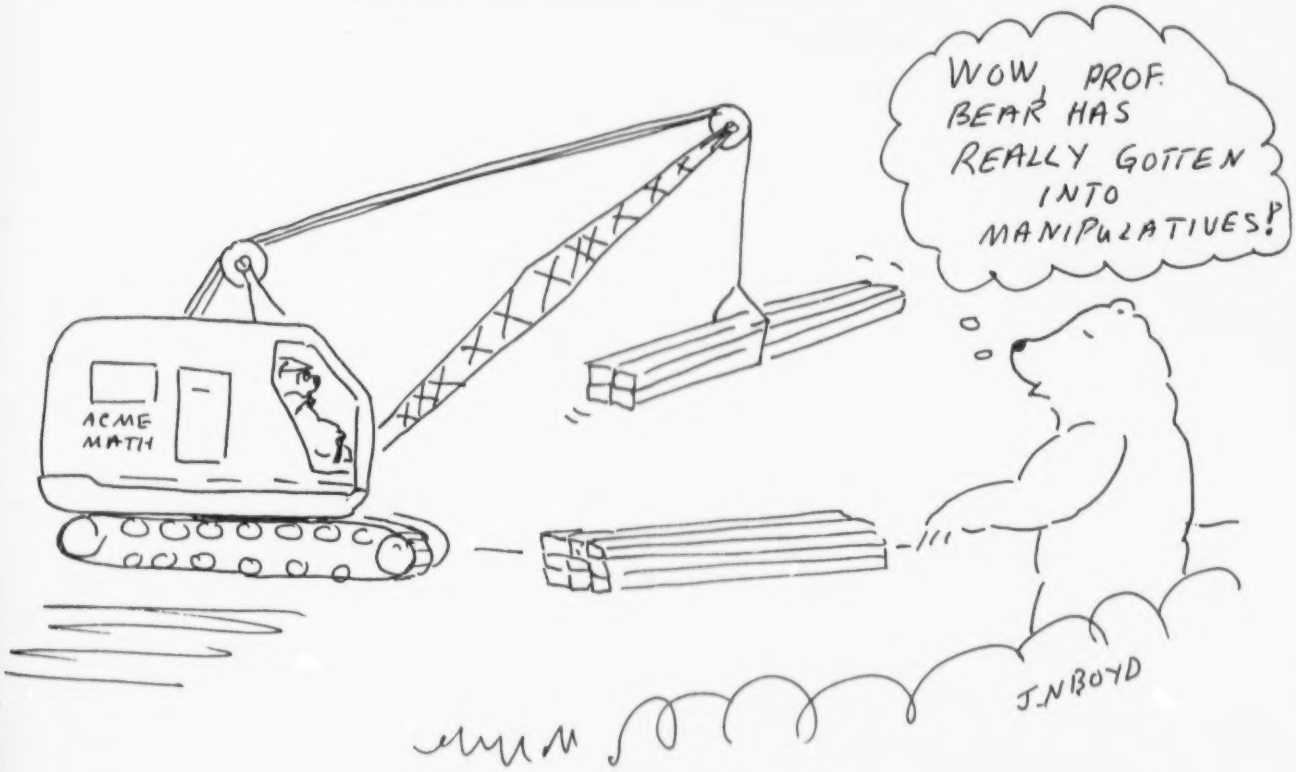
5. **Other functions**
   a. Work with staff in determining needs and priorities for professional development
   b. Assist teachers, on request, in self-evaluation

activities

c. Plan and budget for the purchase of instructional materials
d. Maintain and promote effective communication among grades and schools and with parents, the community, and support personnel
e. Coordinate instruction between regular and special programs to meet individual student needs
f. Manage, schedule, and equip a mathematics laboratory or teacher resource room
g. Provide direct instruction, as appropriate, for individual students, mathematics teams, or special projects

As defined, school mathematics leaders serve as an immediate resource to teachers by demonstrating knowledge, competence, and leadership in mathematics education. Thus, they should know and understand mathematics substantially beyond that which they may be expected to teach, and they should have knowledge and competence in the methods and techniques appropriate for effectively teaching mathematics to elementary/middle school students.

Professor Bear takes the new Standards to heart,



But, perhaps, he has gone a bit too far!

Implementing the Standards
Suggested Staff Development Activities

Liz Hiller

I. Involve the Superintendent and School Board
   A. Send copy of Standards to each member.
   B. Discuss the limited use and effects of norm-referenced tests.
      1. reflects limited range of what students know
      2. emphasizes computation
      3. encourages instrumental understanding
   C. Request financial support for necessary materials.

II. Involve the Curriculum Specialists
   A. Discuss and design programs for the integration of math in other areas of study.
      1. Language Arts - encourage reading, writing, listening, and speaking in Math.
      2. Science - encourage measuring, graphing, estimating, predicting, computation, use of charts.
      3. Social Studies - encourage the use of measuring, ratios, computation, graphing, use of charts.
      4. Physical Education - encourage the use of games which include counting, computation, measurement.

III. Involve the Principals
   A. Send a copy of the new Standards and applicable research.
   B. Discuss tentative plans to encourage teachers to develop activities to implement Standards.
   C. Encourage discussion.

IV. Involve the Teachers
   A. Curriculum Committee
      1. Review current curriculum and make recommendations for changes to better meet the Standards.
         a. evaluate available text, materials.
         b. study the availability of computers and calculators.
   B. Inservice
      1. Learning Activities - use of stories, manipulatives, pictures, and symbols in math.
      2. Manipulatives - how to use everyday items
      3. Learning Centers
   C. Newsletter
      1. Latest research concerning issues
      2. Share ideas of how to implement the Standards
         a. encourage cooperative learning
         b. use real world situations
         c. use of questions which are very similar to those encountered away from school
         d. questioning techniques
   D. Contact Teacher from each School
      1. share ideas with staff
      2. plan school-wide activities, such as Estimation Day
      3. share ideas and needs with specialist

Liz teaches in Arlington, Virginia.

Date and Note Post

Nov. 30-Dec. 2	NCTM Regional Conference in Philadelphia
Nov. 30	American Junior High School Mathematics Examination
Dec. 2	VCTM Board Meeting at Charlottesville
Feb. 3	VCTM Board Meeting at Charlottesville
March 15-17	NCTM Regional Conference in Chattanooga
March 16-17	Annual VCTM Meeting at Charlottesville Contact Person: John Van de Walle, VCU
April 18-21	NCTM Annual Meeting in Salt Lake City
May 5	VCTM Board Meeting at Charlottesville

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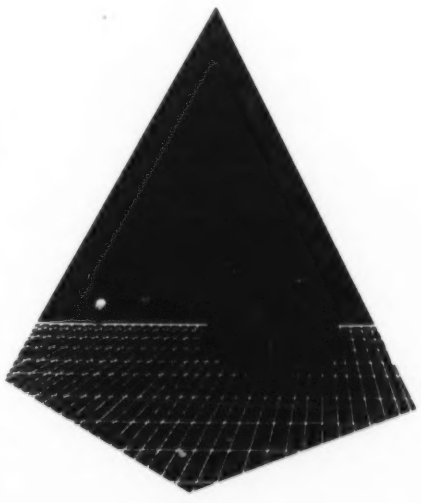

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